When competence hurts: revelation of complex information

Joanna Franaszek, European University Institute

February 2018

Abstract

When the information might be complex and the information processing capacity of economic agents is uncertain, noisy messages do not necessarily indicate bad news. I exploit this intuition to examine a simple sender–receiver persuasion game in which the precise communication of the state of the world depends not only on sender's efforts but also on the state's complexity and the receiver's competence. In this environment the sender-optimal equilibria maximize the amount of noise. The receiver faces a "competence curse" – a smarter type might end up with less information and lower payoff than a receiver with a somewhat smaller competence.

1 Introduction

It is hardly possible to imagine communication between two people that would allow for perfect exchange of any given information. Misunderstanding, misinterpretation or just imprecision might arise due to exogenous frictions, such as the sender's ability to formulate the message, the receiver's competence to absorb and correctly interpret the information content of the message or just the complexity of the matter discussed. Those competence frictions are an inherent feature of real-world communication and there is wide literature regarding such "language barriers" (a term coined by Blume and Board (2006)). However, there is also an endogenous source of frictions coming from possible divergence of interests between the two parties.

Miscommunication may be particularly bothersome if it leads to suboptimal decisions. Between 2006 and 2010, more than a million households in Poland, Croatia, Romania and other Eastern European countries took mortgage loans denominated in Swiss franc, to escape high borrowing costs in their home countries. As the franc had appreciated until 2011, and soared even further in 2015 (when the Swiss National Bank unpegged it from the

I am grateful for valuable comments of Piero Gottardi, Andrea Mattozzi, Andrea Galeotti, Vitor Farinha Luz and Ran Eilat.

euro), the Swiss–franc borrowers were left not only with monthly installments doubled, but also with mortgages worth more than the underlying properties. The dissatisfied borrowers complained about being misinformed, claiming in the European Court of Justice that the bank's presentation "was made in a biased manner, emphasizing the advantages (...), while failing to point out the potential risks or the likelihood of those risks materializing."[1] The Court accentuated that "a term under which the loan must be repaid (...) must be understood by the consumer both at the formal and grammatical level, and also in terms of its actual effects."[2]

The issue of customer's financial (il)literacy became central to the discussion on the unfortunate mortgage holders. Polish data indicates the Swiss–franc borrowers were on *average* relatively wealthy,[3] suggesting their financial literacy might have been relatively high. However, since the banks varied in their loan policies, there is a substantial concern that well-informed risk-lovers were pooled with some risk-averse victims of misinformation.

The mortgage example clearly shows that even when information transmitted between the parties must be truthful, lack of congruence between the receiver and the sender might attenuate communication, if only the latter can manipulate the information content of his message. Anticipating this, the receiver would not only take the strategic incentives into consideration when interpreting the message, but might also find it worthwhile to hide his competence, in order to enhance the informativeness of the sender's message.

Related literature The present article contributes to the growing literature on communication with limited information processing abilities. The main reference is a model by Dewatripont and Tirole (2005), henceforth DT, which inspired the current model. As in their setup, I examine a sender–receiver game, in which the former tries to persuade the latter to take some action. I modify the DT framework by adding another dimension(s) of uncertainty, which is the complexity of the state and the sender's message. Furthermore, I examine how the equilibrium change with the receiver's competence in understanding complex messages and what are his incentives to signal his abilities.

An idea that the receiver has some intellectual, time or attention constraints appeared in a famous model of rational inattention by Sims (2003). Glazer and Rubinstein (2004) derived optimal mechanisms of persuading a receiver that can understand only a single argument. Guembel and Rossetto (2009); Bucher-Koenen and Koenen (2015) define "competence" as the probability of a correct message, similarly to my approach. However, they examine a cheap-talk[4] communication, while I concentrate on truthful information revelation. A close reference is therefore Persson (2017), who builds on the DT framework to examine an

[1](European Court of Justice, 2017, par. 11)

[2]Ibid., par. 51.

[3]National Bank of Poland estimates franc borrowers to have 30% higher annual income and 2.5 higher level of liquid financial assets than the mortgage holders that took a loan in Polish zloty. See National Bank od Poland (2015)

[4]See Crawford and Sobel (1982)

issue of investment in communication with several senders competing for the attention of one receiver – or, in a similar manner – a monopolist sender communicating about several aspects of the good. In her setup, information overload arises endogenously as a result of the receiver's limited attention. If the prior is favorable – i.e. if without communication the receiver took the sender's preferred action – experts send irrelevant cues to prevent the receiver from discovering potentially unfavorable news. I describe a similar equilibrium (however, in a much simpler setup) to focus further on the effect on the receiver's competence. I discover that "smarter" types can end up with a worse outcome.

The result that competence can be "harmful" appeared also in Moreno de Barreda (2010); Ishida and Shimizu (2016); Rantakari (2016) cheap-talk models. They show that the receiver's access to an extra source of information – either after or before the communication takes place – decreases informativeness of the equilibrium messages. Li and Madarász (2008) – also within the cheap-talk framework – show that an extra information about the conflict of interest can decrease communication. However, the mechanism of cheap-talk games is quite different than in my model of information disclosure, where exaggeration is not allowed. Instead of lying, the sender would flood the more competent receiver with more noise. The models of Kessler (1998); Roesler and Szentes (2017) share a similar story, that is, being "too informed" may not be optimal for the receiver. Also, noiseless communication may not be optimal for welfare (see Fishman and Hagerty (1990); Goldstein and Leitner (2013); Blume, Board, and Kawamura (2007)).

Finally, this paper falls into broad literature of truthful, albeit not necessarily complete information transmission. The classic unraveling mechanism, as in Milgrom (1981); Grossman (1981) is disturbed in my model by the presence of uncertainty about the complexity of the state. The setup resembles the one in Shin (1994), who introduces uncertainty about the expert's information space. Similarly, I have uncertainty about the information complexity required to understand the state. When such complexity and the (lack of) competence in communication are introduced, "uninformative" messages may look favorable. This contrasts with the classic Milgrom (1981) result that no news is bad news.

The model also corresponds to a general class of Bayesian persuasion games, described in the seminal paper by Kamenica and Gentzkow (2009). As in Kamenica and Gentzkow (2009); Rayo and Segal (2010); Alonso and Câmara (2013) the sender can benefit from non-full disclosure. The main difference between my approach and Bayesian persuasion models is that I have no commitment on the sender's strategy, much like in Morgan and Stocken (2003); Dziuda (2011).

2 Setup

The model is an augmented version of DT setup, with an additional dimension of uncertainty.[5] There are two players, a sender and a receiver. The receiver is going to choose between a known status-quo that yields payoff (normalized to) 0 to both players, and some risky action A. Action A yields a certain payoff 1 for the Sender and an uncertain payoff that depends on the unknown state of the world ρ for the receiver. The payoff is either ρ_H in state H or ρ_L in state L, with $\rho_H > 0 > \rho_L$. The prior probability of a high state is $\alpha \in (0, 1)$.

The sender has information about the state of the world $\omega \in \{H, L\}$ which he might communicate to the receiver. However, the information may be difficult to transmit. We can imagine e.g. some technical information that requires some expertise to be understood. In particular, the state could be either simple to transmit, which will be denoted by complexity parameter $n = 1$ and happens with probability q or complex with $n = 2$ and prob. $1 - q$.[6] After observing the state realization and its complexity, the sender decides to send a simple or complex message to the receiver. While simple messages – if sufficient – could be understood by any receiver, complex messages require some competence. In particular, an announcement with complexity $m = 2$ could be only understood with probability x, while with probability $1 - x$ the receiver regards the message as noise. I shall call x the receiver's competence and for now assume it is observable by both parties.[7]

The information communicated by the sender must be truthful, but could be noisy - in particular, the sender can exploit the receiver's (lack of) competence by issuing "too complex" message. I shall assume that if the complexity of the message is smaller than the complexity needed to understand the state (i.e. $m < n$), the message is perceived as noise. But also, if a simple state is obfuscated by a complicated announcement, the receiver would only understand it with probability x. Intuitively, simple message about a simple state is always understood perfectly. It is crucial that the sender's announcement does not convey any signal about either the complexity of the state or of the message itself. In particular, if the receiver hears noise, he cannot tell whether it was because of mismatched complexities $(m < n)$ or his own small competence x.

The timing of the model is as follows:

1. Nature chooses:

   (a) state $\omega \in \{H, L\} \sim (\alpha, 1 - \alpha)$

[5] The model is loosely related to the original DT setup, but closely related to the idea mentioned in footnote (32) of Dewatripont and Tirole (2005).

[6] An alternative interpretation would describe n as pieces of information required to understand the state.

[7] I interpret competence as e.g. financial literacy, similarly to Bucher-Koenen and Koenen (2015). Thus, more experienced financial traders simply have higher x. Another, very different idea was employed by Inderst and Ottaviani (2012), where financial literacy was associated with customers' level of strategic "sophistication". In other words, the financial novices were considered to be naive i.e. unaware of the existing conflict of interest.

(b) complexity of the state $n \in \{1,2\} \sim (q, 1-q)$ (known by S)

(c) competence of the receiver $x \in (0,1)$;

2. The sender observes (ω, n) and chooses a truthful message of complexity $m \in \{1,2\}$;

3. The receiver takes action $a \in \{\emptyset, A\}$.

It is crucial to notice that the sender chooses his message complexity m after learning the state of the world (ω, n), in other words, he does not commit to his strategy ex-ante. This is in-line with models of Morgan and Stocken (2003); Dziuda (2011) and in contrast to the important class of Bayesian persuasion models.

Figure 1 summarizes my assumptions regarding the information available to the receiver at each possible triple (ω, n, m). Recall that the source of noise in the message is imperceptible from the receiver's point of view.

There is a plethora of equilibria in the game. To limit myself to some more reasonable cases, I shall assume that any message is in principle cheap, but complex messages are a bit more costly to send. In particular, sending message of complexity $m = 1$ costs 0, while sending message of complexity $m = 2$ costs $c > 0$. Intuitively, c is positive, but very small[8], just enough to induce a choice between strategies that would otherwise be equally preferred by the sender. The assumption reduces the set of equilibria to those where choosing complex messages over simple indeed has some rationale.

3 Equilibrium condition and notation

In order to establish the properties of a perfect Bayesian equilibrium of the game Γ_x with known competence x, one need to specify:

- The sender's message function $\sigma^S : \{H, L\} \times \{1,2\} \to \{1,2\}$, that for every pair (ω, n) (i.e. state and competence needed to understand it) observed by S, defines his message complexity $m \in \{1,2\}$. For notational convenience, I shall describe the sender's strategy as a quadruple: $((H,1),(H,2),(L,1),(L,2)) \mapsto (m_{H1}, m_{H2}, m_{L1}, m_{L2})$.
- The receiver's beliefs μ in each of his information sets (which correspond to messages he understands) $\{H, L, noise\}$.
- the receiver's action function, i.e. $a : \{H, L, noise\} \to \{\emptyset, A\}$.

Lemma 1. *(trivial) In any perfect Bayesian equilibrium the receiver's beliefs upon hearing messages H and L are trivial, i.e.* $\mu(\omega = H|H) = \mu(\omega = L|L) = 1$. *Therefore, his optimal actions are* $a(H) = A$, $a(L) = \emptyset$. *The only nontrivial action is:*

$$a(noise) = \begin{cases} A & \text{if } \mu(H|noise)r_H + \mu(L|noise)r_L \geq 0 \\ \emptyset & \text{otherwise.} \end{cases}$$

[8]The minimal requirement is that $c \ll \frac{1}{2}$.

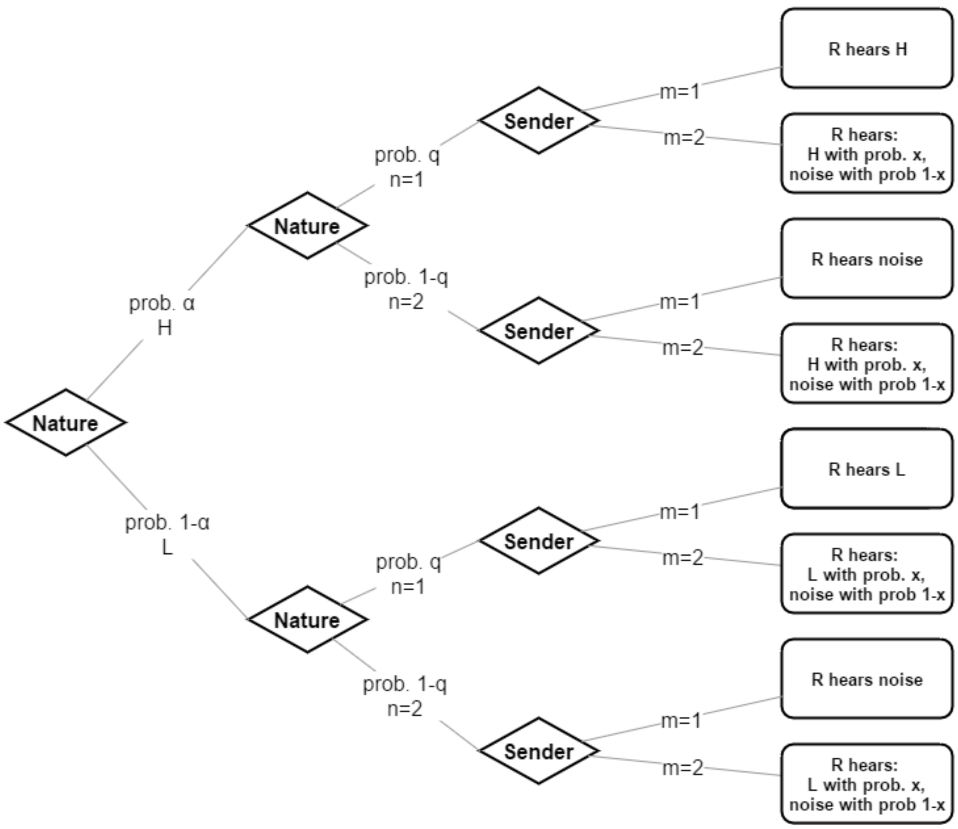


Figure 1: The scheme of information transmission in the game.

Proof. Since the revelation is truthful, upon hearing a non-noisy message the Receiver is certain about the state, which trivially determines his optimal actions. The action upon hearing noise is a result of Bayes rule. Since for any $x < 1$ and any sender's strategy receiving noisy message has positive probability, then for every strategy profile $(m_{H1}, m_{H2}, m_{L1}, m_{L2})$ the receiver can calculate the posterior probability of the state being H when noise was heard. Then, in any perfect Bayesian equilibrium the receiver would take action A upon hearing noise only if according to his Bayesian beliefs: $\mu(H|noise)r_H + \mu(L|noise)r_L \geq 0$. □

Lemma 2. *In any perfect Bayesian equilibrium with costly complex messages, the sender's strategy choice must have $m_{H1} = 1$ and $m_{L2} = 1$.*

Proof. Recall that the Sender chooses his actions already knowing (ω, n). In state $(H, 1)$ the choice of sending simple message, that would surely be understood by the receiver and induce action A, strictly dominates the choice of complex message, that not only is more costly, but also leaves a possibility of misunderstanding. Similarly, in state $(L, 2)$ the complex message is more costly and in no way can it induce a better action than the simple (here meaning: noisy) message. □

Define $\gamma_\alpha = \frac{\alpha}{1-\alpha}$ as the ratio of prior probabilities of states H and L. Note that γ_α as a function of α is strictly increasing and define $\gamma_r = \frac{-r_L}{r_H}$, as the benchmark prior γ_α, that makes the receiver indifferent between action A and the null action. Note that γ_α and γ_r summarize the uncertainty about the state along two different dimensions – while γ_α is directly related to the probability distribution, γ_r depends only on the payoffs. Denote the ratio of the two parameters by $\gamma = \frac{\gamma_\alpha}{\gamma_r} = -\frac{r_H \alpha}{r_L(1-\alpha)}$. Large values of γ mean that the prior is strong (or the gain in the high state is high), while small values of γ indicate that the prior is weak (or the punishment in the low state is substantial). The parameter γ summarizes the gains or losses from uncertainty in the model. Observe that a priori – absent any communication – the receiver would take action A only if $\gamma > 1$.

Theorem 1. *Suppose $0 < c \leq \min(x, 1-x)$. There are four types of perfect Bayesian equilibria of the game with known competence x:*

1. *The informative equilibrium in pure strategies, that exists whenever $\gamma < \frac{1}{1-x}$:*
   (a) *The sender's strategy is $((H,1),(H,2),(L,1),(L,2)) \mapsto (1,2,1,1)$,.*
   (b) *The receiver's beliefs are $\mu(H|noise) = \frac{\alpha(1-x)}{1-\alpha x}$, $\mu(L|noise) = \frac{1-\alpha}{1-\alpha x}$, $\mu(H|H) = \mu(L|L) = 1$,*
   (c) *The receiver's actions are $a(H) = A$, $a(L) = \emptyset$ and $a(noise) = \emptyset$;*
2. *The noisy equilibrium in pure strategies, that exists whenever $\gamma > \frac{1-qx}{1-q}$*
   (a) *The sender's strategy is strategy $((H,1),(H,2),(L,1),(L,2)) \mapsto (1,1,2,1)$,*

(b) *The receiver's beliefs are* $\mu(H|noise) = \frac{\alpha(1-q)}{\alpha(1-q)+(1-\alpha)(1-qx)}$, $\mu(L|noise) = \frac{(1-\alpha)(1-qx)}{\alpha(1-q)+(1-\alpha)(1-qx)}$, $\mu(H|H) = \mu(L|L) = 1$,

(c) *The receiver's actions are* $a(H) = A$, $a(L) = \emptyset$ *and* $a(noise) = A$;

3. *The mixed semi-informative equilibrium for* $\gamma \in \left(\frac{1}{1-x}, \frac{1-qx}{(1-q)(1-x}\right)$

   (a) *The sender's strategy is* $(1, 2, (1-r, r), 1)$ *with* $r = \frac{(1-q)(\gamma(1-x)-1)}{q(1-x)}$,

   (b) *The receiver's actions are* $a(H) = A$, $a(L) = \emptyset$ *and* $a(noise) = (A, \emptyset)$ *with probabilities* $\left(1 - \frac{c}{x}, \frac{c}{x}\right)$;

4. *The mixed semi-noisy equilibrium for* $\gamma \in \left(\frac{1-qx}{1-q}, \frac{1-qx}{(1-q)(1-x}\right)$

   (a) *The sender's strategy is* $(1, (p, 1-p), 2, 1)$ *with* $p = \frac{(1-\gamma(1-q)(1-x)-qx)}{\gamma x(1-q)}$,

   (b) *The receiver's actions are* $a(H) = A$, $a(L) = \emptyset$ *and* $a(noise) = (A, \emptyset)$ *with probabilities* $\left(\frac{c}{1-x}, 1 - \frac{c}{1-x}\right)$;

Corollary 2. *If* $c > max(x, 1-x)$ *then for* $\gamma < 1$ *the only equilibrium is* $\{(1, 1, 1, 1), \emptyset\}$. *For* $\gamma > 1$ *and* $x < c < (1-x)$ *the equilibrium is a mixed–strategy profile* $\{(1, 1, (1-r, r), 1), (b, 1-b)\}$ *with possible corner solution* $r = 1$ *and* $b = 1$ *that is sustained for* $\gamma > \frac{1-qx}{1-q}$. *If* $\gamma > 1$ *and* $1-x < c < x$ *the only equilibrium is a mixed–strategy profile* $\{(1, (p, 1-p), 1, 1), (b, 1-b)\}$ *with possible* $p = 0$ *and* $b = 0$ *whenever* $\gamma > \frac{1}{1-x}$.[9]

Proofs of the theorem and corollary are moved to the Appendix. In line with the intuition about small, almost negligible cost, I shall concentrate in the rest of the paper solely on the case of $c < min(x, 1-x)$. The equilibria in such a case are pictured in the left panel of Figure 2. Notice that even with costly messages, multiple equilibria still persists. For a given pair (γ, x) either the equilibrium is unique, or there are three equilibria (two pure, one mixed or one pure and two mixed).

To make the analysis of the competence effect more explicit, I shall impose an equilibrium selection rule, to enable comparative statics between unique outcomes. Since in my model it is the sender, who possesses the information and therefore has more "initiative", I shall follow the approach of Bayesian persuasion models and concentrate on sender-optimal equilibria.[10] It is not difficult to verify that in fact, the sender-best equilibrium is the one maximizing the amount of noise.

Theorem 3. *The sender-best equilibrium is:*

- $\{(1, 1, 2, 1), a(noise) = A\}$ *for* $\gamma > \frac{1-qx}{1-q}$,
- $\{(1, 2, 1, 1), a(noise) = \emptyset\}$ *for* $\gamma < \min\left(\frac{1-qx}{1-q}, \frac{1}{1-x}\right)$,
- $\{(1, 2, (1-r, r), 1), a(noise) = (1 - \frac{c}{x}, \frac{c}{x})\}$ *for* $\frac{1}{1-x} < \gamma < \frac{1-qx}{1-q}$.

[9]The case $c > max(x, 1-x)$ is ruled out, since the minimal requirement is $c < 1/2$.

[10]Glazer and Rubinstein (2012) propose a different approach, where the receiver commits to a "persuasion codex". With such an assumption, the selected equilibria would be those optimal for the receiver.

The noisy equilibrium $\{(1,1,2,1), A\}$, is preferred by the sender, whenever it can be supported by the receiver's beliefs. If the noisy equilibrium fails to exist, the existing informative or mixed semi-informative equilibrium is unique, therefore, it is sender-best. Sender-best equilibria are pictured in the right panel of Figure 2.

4 Receiver's expected gain in the game

We may now analyze the receiver's gain from the game and how it depends on his language competence x. Assume $1 < \gamma < \frac{1}{1-q}$, which is the most interesting range, as depending on x, there is a possibility of up to three different (sender-best) equilibria. Notice that for a given γ, a higher competence may be a burden for the receiver, as it might result in a noisy equilibrium $\{(1,1,2,1), A\}$, while for somewhat lower values of x the unique equilibrium is either informative $\{(1,2,1,1), \emptyset\}$ or semi-informative $\left\{(1,2,(1-r,r),1), \left(\frac{c}{1-x}, 1-\frac{c}{1-x}\right)\right\}$.

Effect of competence

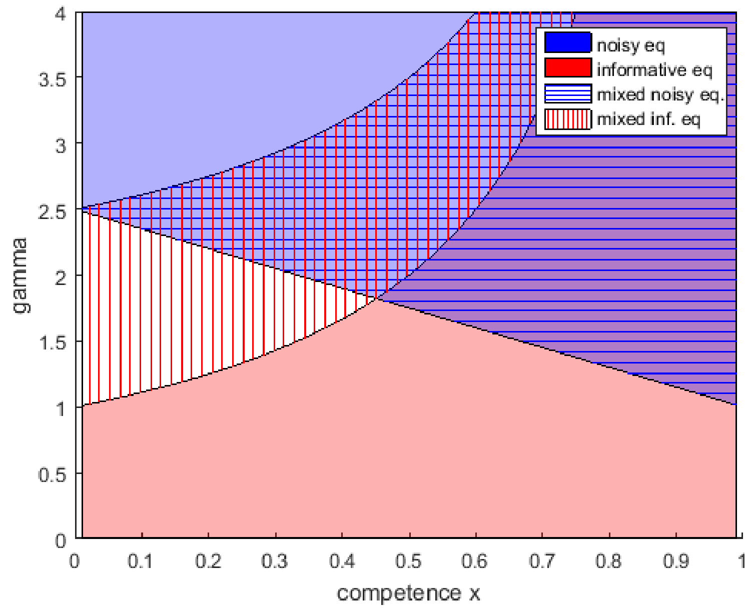
To see why higher types face a "competence curse", observe that for more competent receivers, noise might be a favorable message. Assume that the sender chooses a strategy $(1,1,2,1)$ and the receiver anticipates it. In a state $(L,1)$, the receiver hears a signal that he correctly understands as L with probability x. High values of x are a sign of competence, therefore the receiver is "relatively good" in identifying low state correctly with certainty. As a result he believes noise to be less likely to arise in the low state. As $m(L|noise)$ is low, $m(H|noise)$ must be relatively high, thus, the noise becomes a signal of a good state. Competence becomes a curse – because of the receiver's good understanding of low states, the sender is able to "sell" the noisy message as a favorable signal and maintain the equilibrium in which very little information is transmitted.

More formally, let us analyze the receiver's expected payoff:

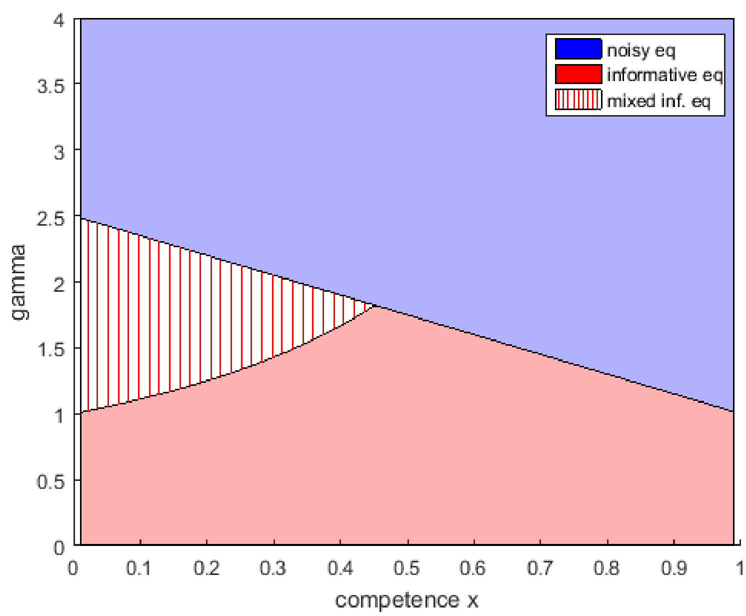
$$\begin{aligned} E^R_{info\,eq}\rho(x) &= (\alpha q + \alpha(1-q)x)\,\rho_H \\ E^R_{noisy\,eq}\rho(x) &= \alpha\rho_H + \rho_L(1-\alpha)(1-qx) \end{aligned} \tag{1}$$

For a given equilibrium profile the receiver always benefits larger competence – as both $E^R_{info\,eq}\rho(x)$ and $E^R_{noisy\,eq}\rho(x)$ are increasing in x. However, this is not the case when a change in x would induce a change in the equilibrium profile.

Assume (x,γ) result in a noisy equilibrium and consider a downward change in x. As competence decreases, $m(H|noise)$ – which is an increasing function of x in the noisy equilibrium – also plummets, up to a point where it is no longer profitable for the receiver to choose A upon hearing noise and he would rather take the $\emptyset$ action instead. The sender is then forced to switch to an informative strategy $(1,2,1,1)$ and a new equilibrium arises. It must be noted that a switch from a noisy to informative equilibrium – as x decreases –



(a) All equilibria



(b) Sender-best equilibria

Figure 2: Pure (solid fill) and mixed (pattern fill) equilibria in the game for a given γ and x.

not only increases informativeness, but brings a discontinuous jump in receiver's utility (see Figure 3).

Take $x' = \frac{1}{q}(1-\gamma(1-q))$ and let us analyze the expected gain/loss for an equilibrium switch when x decreasing around a neighborhood of x':

$$Er(x \searrow x') - Er(x \nearrow x') = -\alpha\rho_H(1-q)x' < 0$$

A marginal change in x around the threshold bring a discrete decrease in utility. Therefore, for a small ϵ, two receiver types $x' + \epsilon$ and $x' - \epsilon$ not only end up in different equilibria, but also the more competent receiver is strictly worse off. If it was possible, he would rather decrease his competence to $x' - \epsilon$ to induce an informative equilibrium than remain more competent, but less informed.

Similar reasoning applies to a switch from a noisy to mixed semi-informative equilibrium, whenever γ is so high that the informative equilibrium no longer exists. Observe that by a definition of a mixed equilibrium, the receiver is indifferent between his two choices conditioned on noise, therefore:

$$E^R_{mixed(p,r,\beta)}\rho = \alpha\rho_H\left(q + (1-q)x\right)$$

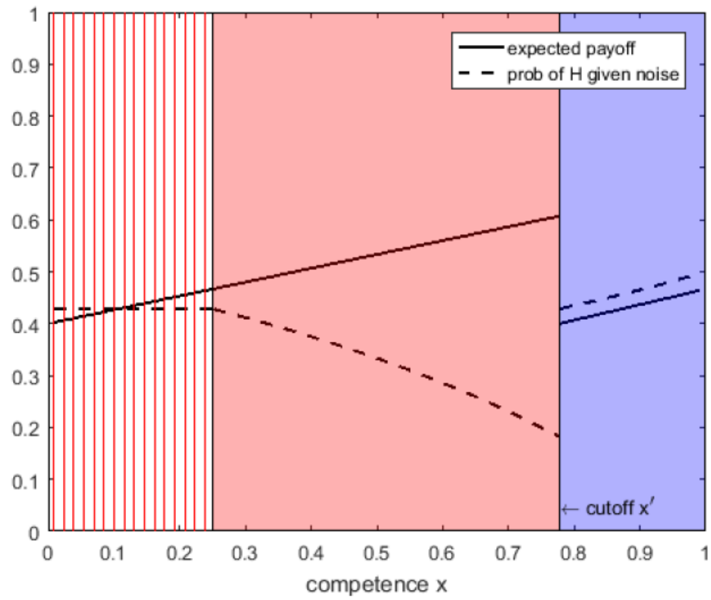
, which is of the same functional form as $E^R_{info\,eq}\rho$. Therefore, also a switch from a noisy to a semi-informative mixed equilibrium brings a discontinuous loss in the receiver's utility and the receiver type close to the threshold would prefer to compromise some competence in order to get a better outcome.

However, when the equilibrium is switched from an informative to mixed semi-informative, the change in utility is continuous and the utility is increasing in x. In fact, since the receiver's utility in both equilibria share the same functional form, the semi-informative equilibrium is a natural "alternative" to an informative equilibrium, whenever the latter cannot be sustained. Therefore, the receiver of type $x \in [0, x']$ has no incentive to reduce his competence, even if it was possible.

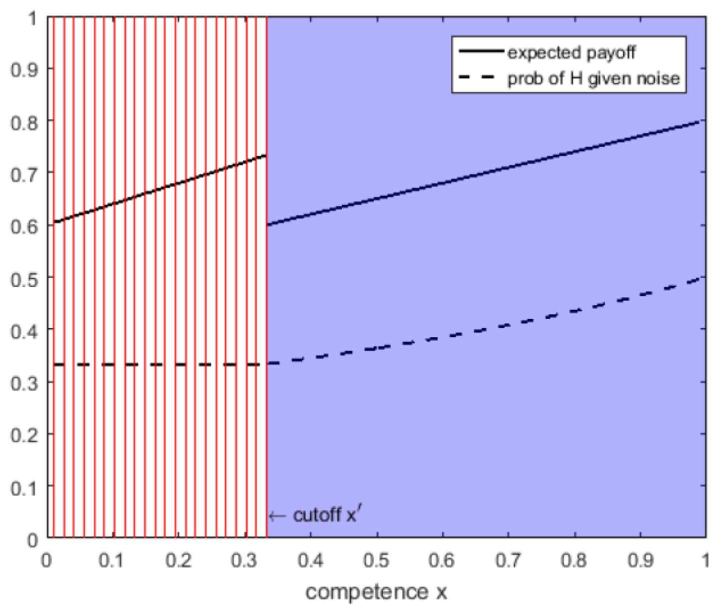
Change in a prior signal

Similar reasoning applies to changes in γ. Just like with competence, having higher initial prior does not necessarily benefit the receiver. In particular, if the receiver faces an increase in γ, he might end up in worse equilibrium. This is quite intuitive, as more favorable prior makes the receiver more likely to choose A, thus decreasing the sender's incentives to transmit information.

The prior information represented by γ and the communication competence x are substitutes. It would be interesting – but beyond the scope of this paper – to examine a model in which the two types of communication skills are substantially different; while one dimension represents the stock of knowledge, the other describes the ability to absorb



(a) $\rho_H = 4/3,\ \rho_L = -1,\ q = 0.6,\ \alpha = 0.5$



(b) $\rho_H = 2,\ \rho_L = -1,\ q = 0.6,\ \alpha = 0.5$

Figure 3: Discrete change in the receiver's expected utility and $m(H|noise)$ when the equilibrium changes from the informative (red) or semi-informative (red stripes) to noisy (blue) profile.

new knowledge. In reality, those two dimensions of informational literacy are distinct skills.

Private information about competence

I have shown that the receiver might face a "competence curse" – in particular, if his competence is so high that it induces an uninformative equilibrium, the receiver might be worse off that with somewhat lower x. However, reducing x is hardly possible.

Assume that competence becomes the receiver's private information. To simplify, let us consider a case in which competence may be either x_L with probability π or x_H with probability $1-\pi$ and denote the equilibrium probability of type i choosing A upon hearing noise as b_i. To make things interesting, assume $x_L < \bar{x} < x_H$, where $\bar{x}$ satisfies $\frac{1}{1-\bar{x}} = \frac{1-q\bar{x}}{1-q}$[11] and the probabilities π or $1-\pi$ are sufficiently separated from 0 and 1 – so that the two–types case does not trivially collapse to the one–type setup.

It can be shown (see: Appendix) that even though there are multiple equilibria in the setting, as long the noisy profile $\{(1,1,2,1), b_L = 1, b_H = 1\}$ can be sustained – that is for $\gamma > \frac{1-qx_L}{1-q}$ – it remains the sender–best equilibrium. If $\gamma < \frac{1}{1-x_L}$ the unique equilibrium is the informative one in which neither receiver type chooses A unless he is certain about the state. The only interesting case is therefore $\frac{1}{1-x_L} < \gamma < \frac{1-qx_L}{1-q}$. Indeed, in this range the high type might benefit from the uncertainty. The only (therefore, sender-best) equilibrium is $\{(1,2,(1-r,r),1), b_L = \frac{c}{\pi(1-xH)}, b_H = 0\}$ with $r \in [0, \bar{r}(x_L)]$.[12] Notice that the high type is not only better off than without uncertainty about x but also his outcome is higher than the low type's payoff. If the sender attempts to persuade the low type, he must send at least a semi-informative message. More competent type "freerides" and can now enjoy the more favorable outcome. The low type, on the other hand, enjoys the same outcome as if he played single-handedly.

This outcome would persist if the receiver could send a cheap-talk message. Notice that the high type would always want to send the same message as the low type, as it is in his best interest to be pooled. If instead the types would be able to credibly certify their types at no cost, the low type is able to separate, but has no strong incentives to do so – in fact, he is indifferent between being certified or not.

5 Summary

In this paper, I examine a persuasion game in which the state of the world might be difficult to transmit. The sender perfectly observes the state of the world and its complexity, i.e. how difficult it is to understand the state. The sender then chooses a simple or a complex message, with the latter bearing a small cost. His goal is to persuade the receiver to take

[11] $\bar{x}$ is a crossing point of the indifference curves that define equilibrium conditions

[12] More specifically, $\bar{r}(x_L) = \frac{q-x_L-qx_L(1-x_L)}{q(1-xL)}$.

some action A that yields the latter an uncertain payoff. As an alternative, the receiver can take an outside option $\emptyset$ with a payoff 0.

I show that when there is uncertainty about the complexity of information, noise is no longer perceived purely as ”bad news”. This is because noise might come from two different sources: exogenous complexity of information required for successful communication or endogenous sender's incentive to obfuscate the unfavorable state.

I concentrate on sender-best equilibria and show that there are three types of equilibrium profiles. If the prior γ is high, the receiver is willing to choose action A upon hearing noise and the sender can sustain his mostly preferred noisy equilibrium in which little information is transmitted. If γ is small, the receiver is more wary and the sender's best option is to send as much information as possible and refrain from issuing extra noise. Depending on (γ, x), this leads to either an informative or a semi-informative equilibrium. The surprising result, however, is that for a given γ more competent receiver types might end up in the worse, noisy equilibrium than somewhat smaller types, who are guaranteed to end up in the informative outcome. The competence becomes a curse.

To understand the result, suppose the sender tries to 'sell' noise as a good signal and issue intentionally complex messages. Since information revelation must be truthful, the announcements are sometimes correctly understood – more often, if the receiver is more competent. As a result, upon hearing noise the high type would attach less likelihood to the state being low than high. Thus, noise becomes a favorable message and the sender has no incentive to transmit any information. This equilibrium cannot be sustained for a less competent receiver, precisely because of his little understanding of the complex messages. The incompetent type is warier and unwilling to choose A upon hearing noise. Therefore, the sender has no choice, but to persuade him with an informative announcement.

In a comparative statics exercise I show that the utility loss associated with an equilibrium change is discrete and negative – in other words, the smart receiver would have a strong incentive to ”play dumb”. While in the standard setup, this is not possible, I also examine a game in which the sender is uncertain about the receiver's competence, which might be either high or low. I show that for a relevant range of γ the competent receiver strictly benefits from extra uncertainty, as he now ends up in the unique semi-informative outcome. The low type's outcome is the same, so he has no strict incentive to disturb the pooling equilibrium.

Appendix

Proof of theorem 1

Proof. I shall analyze which strategy profiles might arise in an equilibrium. By Lemma 2, there are only four feasible strategies of the sender: $(1,1,1,1)$, $(1,2,1,1)$,$(1,1,2,1)$, $(1,2,2,1)$. Notice also that in any perfect Bayesian equilibrium the receiver's beliefs must be consistent with the sender's strategy. Let us assume $c < \min(x, 1-x)$, which means there is at least some incentive to invest in costly message. Examine four cases:

1. The sender uses strategy $(1,1,1,1)$. Such strategy is consistent with the receiver's beliefs $\mu(H|noise) = \alpha$ and $\mu(L|noise) = 1-\alpha$. Assume that the receiver chooses $a(noise) = A$. The sender might benefit from deviating to $(1,1,2,1)$, generating noise with positive probability Assume the receiver takes an action $\emptyset$ when hearing noise. The sender can benefit from deviating to $(1,2,1,1)$, i.e. more informative message. Thus strategy $(1,1,1,1)$ is never optimal.

2. The sender uses strategy $(1,2,1,1)$. This strategy is consistent with the receiver's beliefs $\mu(H|noise) = \frac{\alpha(1-x)}{1-\alpha x}$ and $\mu(L|noise) = \frac{1-\alpha}{1-\alpha x}$. Assume the receiver takes A when hearing noise. Then the sender has an incentive to deviate to a more noisy message $(1,2,2,1)$. In the other case, when the receiver takes $\emptyset$ upon hearing noise, there is no incentive to deviate. The appropriate beliefs imply $\gamma < \frac{1}{(1-x)}$ and in such a case the strategies $\{(1,2,1,1),\ a(noise) = \emptyset\}$ constitute an equilibrium.

3. The sender uses strategy $(1,1,2,1)$. Upon hearing noise the receiver would take action A if $\gamma \geq \frac{1-qx}{1-q}$ and $\emptyset$ otherwise. In the latter case, i.e. $a(noise) = \emptyset$, the sender has an incentive to deviate from costly $(1,1,2,1)$ to less costly $(1,1,1,1)$. If $a(noise) = A$, there is no incentive to deviate and the profile $\{(1,1,2,1), a(noise) = A\}$ constitutes an equilibrium when $\gamma \geq \frac{1-qx}{1-q}$.

4. The sender uses strategy $(1,2,2,1)$. Upon hearing noise the receiver would take action A if $\gamma \geq \frac{1-qx}{(1-q)(1-x)}$ and $\emptyset$ otherwise. If $a(noise) = A$ the sender has an incentive to deviate from more costly $(1,2,2,1)$ to less costly $(1,1,2,1)$. In the second case, when $a(noise) = \emptyset$, the sender has an incentive to deviate from costly $(1,2,2,1)$ to less costly $(1,2,1,1)$. Thus, $(1,2,2,1)$ is not used in any equilibrium.

Notice that the analysis above could be also performed taking purely interim point of view, i.e. analyzing just the actual choice in critical states $(H,2)$ and $(L,1)$. This approach would be used to examine mixed strategies. Since the choice is made after the state is realized, the choices of m_{H2} and m_{L1} are interdependent only through the beliefs they induce in the equilibrium. The mixing could be an arbitrary $(1,(p,1-p),(1-r,r),1)$. In any mixed equilibrium in which at least one of p, r is interior, the receiver must be indifferent between choosing A and $\emptyset$, therefore p, r must satisfy:

$$\gamma = \frac{qr(1-x) + (1-q)}{(1-q)(1-x+px)}. \tag{2}$$

The receiver's response is $(b, 1-b)$ where $b = P(a(noise) = A)$.

Assume the receiver plays according to a strategy $(b, 1-b)$ with $b \in (0,1)$. Consider the state $(H, 2)$ and the sender's choice of $(p, 1-p)$ that costs $c(1-p)$. Notice that the sender's payoff is linear in p.

$$E(\text{payoff in}(H,2)) = (-x(1-b)+c)p + \alpha(1-q)(x+(1-x)b-c) \tag{3}$$

If $b = 1 - \frac{c}{x}$ then the sender's choice of p could be arbitrary, as the payoff is constant in p. If, $b < 1 - \frac{c}{x}$ then the sender finds it optimal to choose $p = 0$ and if $b > 1 - \frac{c}{x}$ then the optimal choice is $p = 1$.

A similar reasoning applies to changes in r, when the state is $(L, 1)$ The strategy $(1-r, r)$ costs cr and any incremental change in r results in a change in utility:

$$E(\text{payoff in}(L,1)) = ((1-x)b - c)r \tag{4}$$

Notice that generically for a given pair (c, x) it cannot simultaneously hold that $b = 1 - \frac{c}{x}$ and $b = \frac{c}{1-x}$ as long as $c \neq x(1-x)$. Therefore, at most one of (3) and (4) can be independent of p or r and allow for an interior choice of the parameter. Therefore mixing would be performed only in one of the critical states $(H, 2)$ and $(L, 1)$. In the other state, the incentives would drive the receiver to choose a corner solution from a set $\{0, 1\}$. This is quite clear if we observe that the sender's decision is indeed a linear programming problem.

The first type of mixed equilibrium is of the form $\{(1, (p, 1-p), 2, 1), (b_1, 1-b_1)\}$ with $b_1 = 1 - \frac{c}{x}$ and exists whenever $\frac{1-qx}{1-q} \leq \gamma \leq \frac{1-qx}{(1-q)(1-x)}$. Notice that for small c, the probability of the receiver taking action upon hearing noise is close to 1, therefore this equilibrium is relatively noisy We shall call it a semi-noisy mixed equilibrium.

The second type of mixed equilibrium is of the form $\{(1, 2, (1-r, r), 1), (b_2, 1-b_2)\}$ with $b_2 = \frac{c}{1-x}$ and exists whenever $\frac{1}{(1-x)} \leq \gamma \leq \frac{1-qx}{(1-q)(1-x)}$. Whenever c is small, b_2 is close to zero. Therefore this equilibrium would be labeled as a semi-informative mixed equilibrium.

In any mixed equilibrium, the condition (2) must be satisfied, thus the mixed equilibria can be sustained only within some subset of the (x, γ)-space.

In the unlikely case of $c = x(1-x)$, the equilibrium is $\{(1, (p, 1-p), (1-r, r), 1), (x, 1-x)\}$. In this equilibrium the probability of the receiver accepting the action A is exactly equal to his competence.

$\square$

Proof of corollary 2

Proof. The result in a corollary follows naturally from the proof above. For $c > 1 - x$ the sender does never have an incentive to choose any r but 0, as is clear from his payoff characterization in (2.4). A strategy profile $(1, (p, 1-p), 1, 1)$ could be supported as long as γ satisfies (2.2) for some $p \in [0, 1\ ]$, which implies $1 \leq \gamma \leq \frac{1}{(1-x+px)}$. For $\gamma < 1$ the equilibrium is $\{(1, 2, 1, 1), A\}$ and for $\gamma > 1/(1-x)$ it must be $\{(1, 1, 1, 1), \emptyset\}$. For $c > x$ the sender with incentives like in (2.3) must choose $p = 1$. A mixed strategy $(1, 1, (1-r, r), 1)$ is feasible as long as $1 \leq \gamma \leq \frac{1-qx}{1-q}$. For $\gamma < 1$ the equilibrium is $\{(1, 1, 1, 1), \emptyset\}$ and for $\gamma > (1-qx)/(1-q)$ it must be $\{(1, 1, 2, 1), A\}$. □

Proof of theorem 3

Proof. This result is quite intuitive, but I will prove it formally Notice that the sender's payoff from an arbitrary pure or mixed strategy of the general form $\{(1, (p, 1-p), (1-r, r), 1)\ , (b, 1-b)\}$ is:

$$\begin{aligned}Eu^S(\text{eq. profile}) = \alpha q + \alpha(1-q)[(1-p)(x+(1-x)b) + pb] + \\ + (1-\alpha)qr(1-x)b + (1-\alpha)(1-q)b - c(\alpha(1-q)(1-p) + r(1-\alpha)q).\end{aligned}$$

The mixed strategy payoff is quite easy to derive. For mixed equilibria, recall that by the definition of equilibrium b, the payoff must be independent of p in a semi-noisy equilibrium and of r in the semi-informative equilibrium.

$$\begin{aligned}Eu^S(\text{info eq.}) &= \alpha q + \alpha(1-q)(x-c), \\ Eu^S(\text{noisy eq.}) &= \alpha + (1-\alpha)((1-qx) - cq), \\ Eu^S(\text{semi-info eq.}) &= \alpha q + \alpha(1-q)[x + (1-x)b - c] + (1-\alpha)(1-q)b, \\ Eu^S(\text{semi-noisy eq.}) &= \alpha q + \alpha(1-q)[x + (1-x)b - c] + (1-\alpha)[q((1-x)b - c) + (1-q)b].\end{aligned}$$

It is clear that $Eu^S(\text{noisy eq.}) > Eu^S(\text{info eq.})$ as $x - c < x + c < 1$ for $c < \min(x, 1-x)$. Notice also that the mixed profiles are increasing in b, therefore:

$$\begin{aligned}Eu^S(\text{semi-noisy eq.}) < \alpha q + \alpha(1-q)(1-c) + (1-\alpha)(1-qx-qc) < Eu^S(\text{noisy eq}) \\ Eu^S(\text{semi-info eq.}) < \alpha q + \alpha(1-q)(1-c) + (1-\alpha)(1-q) < Eu^S(\text{noisy eq})\end{aligned}$$

Therefore $Eu^S(\text{noisy eq.})$ dominates all other payoffs. □

Equilibria in the game with two receiver types

As an additional comment, I shall describe the equilibria in the game of one sender playing against a receiver of uncertain competence that is either x_L with probability π or x_H with probability $1 - \pi$.

The receiver of type i chooses a strategy $(b_i, 1-b_i)$ with $b_i \in [0,1\,]$. The sender plays a profile $(1,(p,1-p),(1-r,r),1)$ that also incorporates pure strategies.

The sender chooses his actions after learning the state, so again, we will consider his choices in the crucial states $(H,2)$ and $(L,1)$. In the high and complex state, he would prefer to send a complex message than a simple if:

$$\pi x_L(1-b_L) + (1-\pi)x_H(1-b_H) \geq c \tag{5}$$

In $(L,1)$ the complex message is preferred as long as:

$$\pi(1-x_L)b_L + (1-\pi)(1-x_H)b_H \geq c \tag{6}$$

Denote $f(p,r,x) = \frac{qr(1-x_i)+1-q}{(1-q)(1-x_i+px_i)}$. The receiver chooses $b_i = 0$ if $\gamma < f(p,r,x_i)$, $b_i = 1$ if $\gamma > f(p,r,x_i)$ and $b_i \in (0,1)$ if $\gamma = f(p,r,x_i)$. Notice that $f(p,r,x)$ is generically not constant, so typically at most one x_i may satisfy $f(p,r,x_i) = \gamma$, and thus have $b_i \in (0,1)$. This leaves us with eight possible cases:

- $b_L = 0, b_H = 0$. The sender's strategy must be $(1,2,1,1)$ and $\gamma < \frac{1}{1-x_H}$;
- $b_L = 0, b_H \in (0,1)$. Then $b_H = \frac{c}{(1-\pi)(1-x_H)}$ and the sender responds with $(1,2,(1-r,r),1)$ which can be sustained if $\gamma = \frac{qr(1-x_H)+1-q}{(1-q)(1-x_H)}$. But $f(p,r,x)$ increasing, so it can't be that $\gamma \geq f(p,r,x_L)$, that is required for $b_L = 0$. Such an equilibrium does not exist.
- $b_L = 0, b_H = 1$. Then $(1,2,2,1)$, but we can't have $\gamma \leq \frac{1-qx_H}{(1-q)(1-x_H)}$ and $\gamma \geq \frac{1-qx_H}{(1-q)(1-x_H)}$, as $f(p,r,x)$ increasing. Again, not feasible.
- $b_L \in (0,1), b_H = 0$. Then $b_L = \frac{C}{\pi(1-x_H)}$, the sender responds with $(1,2,(1-r,r),1)$ and $\gamma = \frac{qr(1-x_L)+(1-q)}{(1-q)(1-x_L)} < \frac{qr(1-x_H)+(1-q)}{(1-q)(1-x_H)}$.
- $b_L \in (0,1), b_H = 1$. Then $b_L = 1 - \frac{c}{\pi x_L}$, the sender responds with $(1,(p,1-p),2,1)$ and $\gamma = \frac{1-qx_L}{(1-q)(1-x_L+px_L)} > \frac{1-qx_H}{(1-q)(1-x_H+px_H)}$, which might be sustained only if $p > 1-q$;
- $b_L = 1, b_H = 0$. Then the sender responds with $(1,2,2,1)$ and now it's possible that $\frac{1-qx_H}{(1-q)(1-x_H)} > \gamma > \frac{1-qx_L}{(1-q)(1-x_L)}$.
- $b_L = 1, b_H \in (0,1)$. Then $b_H = 1 - \frac{c}{(1-\pi)x_H}$ and $(1,\ (p,\ 1-p),2,1)\,\frac{1-qx_H}{(1-q)(1-x_H+px_H)} = \gamma > \frac{1-qx_L}{(1-q)(1-x_L+px_L)}$ and must be $p < 1-q$;
- $b_L = 1, b_H = 1$. The sender chooses $(1,\ 1,2,\ 1)$ and $\gamma > \frac{1-qx_L}{(1-q)} > \frac{1-qx_H}{(1-q)}$;

For a given γ, a few equilibria might coexist. However, as is clear from the proof of theorem 3 above, whenever the noisy equilibrium exists, it dominated (from the sender's point of view) all other possibilities. This leaves us with only three sender-best equilibria:

- $\{(1,1,2,1),\ b_L = 1,\ b_H = 1\}$ whenever $\gamma > \frac{1-qx_L}{(1-q)}$;
- $\{(1,2,\ (1-r,r),\ 1),\ b_L = \frac{c}{\pi(1-x_H)},\ b_H = 0\}$ whenever $\frac{1}{1-x_L} < \gamma < \frac{1-qx_L}{(1-q)}$ with $r = \frac{(1-q)(\gamma(1-x)-1)}{q(1-x)}$;
- $\{(1,2,1,1),\ b_L = 0,\ b_H = 0\}$ whenever $\gamma < \frac{1}{1-x_L}$;

Notice that those equilibria are the same *as if* the sender has played only against a low-type receiver. The low-type receiver obtains exactly the same outcome as in a game without the presence of a high type. However, x_H's payoff is substantially different. By being pooled with the low type in an informative (or semi-informative) equilibrium, the competent receiver is able to avoid being lured into a noisy equilibrium.

References

Alonso, R., and O. Câmara (2013): "Persuading skeptics and reaffirming believers," Discussion paper, University of Southern California, Marshall School of Business.

Blume, A., and O. J. Board (2006): "Language barriers," *Econometrica*, 12(13), 16–17.

Blume, A., O. J. Board, and K. Kawamura (2007): "Noisy talk," *Theoretical Economics*, 2, 395–440.

Bucher-Koenen, T., and J. Koenen (2015): "Do Seemingly Smarter Consumers Get Better Advice?," Discussion paper.

Crawford, V. P., and J. Sobel (1982): "Strategic information transmission," *Econometrica*, 50(6), 1431–1451.

Dewatripont, M., and J. Tirole (2005): "Modes of communication," *Journal of Political Economy*, 113(6), 1217–1238.

Dziuda, W. (2011): "Strategic argumentation," *Journal of Economic Theory*, 146(4), 1362–1397.

European Court of Justice (2017): "Ruxandra Paula Andriciuc and others v Banca Românească SA," Judgement of 20 September 2017, C-186/16, EU:C:2017:703.

Fishman, M. J., and K. M. Hagerty (1990): "The optimal amount of discretion to allow in disclosure," *The Quarterly Journal of Economics*, 105(2), 427–444.

Glazer, J., and A. Rubinstein (2004): "On optimal rules of persuasion," *Econometrica*, 72(6), 1715–1736.

——— (2012): "A Model of Persuasion with Boundedly Rational Agents," *Journal of Political Economy*, 120(6), 1057–1082.

Goldstein, I., and Y. Leitner (2013): "Stress Tests and Information Disclosure," Discussion paper.

Grossman, S. J. (1981): "The Informational Role of Warranties and Private Disclosure about Product Quality," *The Journal of Law and Economics*, 24(3), 461.

Guembel, A., and S. Rossetto (2009): "Reputational cheap talk with misunderstanding," *Games and Economic Behavior*, 67, 736–744.

Inderst, R., and M. Ottaviani (2012): "Financial advice," *Journal of Economic Literature*, 50(2), 494–512.

Ishida, J., and T. Shimizu (2016): "Cheap talk with an informed receiver," *Economic Theory Bulletin*, 4(1), 61–72.

Kamenica, E., and M. Gentzkow (2009): "Bayesian Persuasion," *American Economic Review*, 101(October), 2590–2615.

Kessler, A. S. (1998): "The Value of Ignorance," *The RAND Journal of Economics*, 29(2), 339–354.

Li, M., and K. Madarász (2008): "When mandatory disclosure hurts: Expert advice and conflicting interests," *Journal of Economic Theory*, 139(1), 47–74.

Milgrom, P. (1981): "Good news and bad news : representation theorems and applications," *The Bell Journal of Economics*, 12(2), 380–391.

Moreno de Barreda, I. (2010): "Cheap talk with two-sided private information," Discussion paper, London School of Economics.

Morgan, J., and P. C. Stocken (2003): "An analysis of stock recommendations," *RAND Journal of Economics*, 34(1), 183–203.

National Bank od Poland (2015): "Zasobność gospodarstw domowych w Polsce," .

Persson, P. (2017): "Attention Manipulation and Information Overload," *NBER Working Papers Series*, 23823.

Rantakari, H. (2016): "Soliciting Advice: Active versus Passive Principals," *Journal of Law, Economics, and Organization*, 32(4), 719–761.

Rayo, L., and I. Segal (2010): "Optimal Information Disclosure," *Journal of Political Economy*, 118(5), 949–987.

Roesler, A. K., and B. Szentes (2017): "Buyer-optimal learning and monopoly pricing," *American Economic Review*, 107(7), 2072–2080.

Shin, H. S. (1994): "The Burden of Proof in a Game of Persuasion," *Journal of Economic Theory*, 64(1), 253–264.

Sims, C. A. (2003): "Implications of rational inattention," *Journal of Monetary Economics*, 50(3), 665–690.