Demand

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Outline:

Why are we concerned with demand systems?

- Information on incentives for firms influencing
  - pricing decision
  - alternative investments (new products, advertising, ect.)
- Welfare analysis
  - consumer surplus gains to product introductions
  - impact of regulatory policies
  - construction of *ideal price indices*

Why are we concerned with demand systems? – cont.

- Demand systems are the major tool for comparative static analysis of any change in a market that does not have an immediate impact in costs
- Estimation of price elasticities are important for:
  - Marketers
    - in designing pricing policies
    - introducing new goods
  - Policy officials
    - in designing tax schemes
    - in evaluating mergers

The Consumer's Problem

- We assume the consumer's preference relation is represented by a utility function u, that is differentiable, strictly increasing, and strictly quasiconcave
- The feasible set $B = \left[\mathbf{q} \in R^n_+ | \mathbf{p} \cdot \mathbf{q} \leqslant y\right]$, where $y > 0$ and $\mathbf{p} > 0$
- We assume that the consumer chooses the alternative from her budget set that is best according to her preferences
- Choose an alternative from the budget set that maximizes utility subject to the constraint that total expenditure does not exceed income
- The consumer chooses a $\mathbf{q}^*$ that solves $max_{q \in R^n_+} u(\mathbf{q})$ s.t. $\mathbf{p} \cdot \mathbf{q} \leqslant y$

Consumer's problem: Questions

- Is there a solution?
  - Yes. Weierstrass theorem
- Is the solution unique?
  - Yes. Because B is convex and the utility function has been assumed to be strictly quasiconcave
- How do we find the solution?

Characterization of the Solution

- Define the Lagrangian $L(q;\lambda) := u(\mathbf{q}) - \lambda(\mathbf{p}\cdot\mathbf{q} - y)$
- Necessary and sufficient condition for $\mathbf{q}^* > 0$ to solve the consumer's problem is that there exists $\lambda^* \geqslant 0$ such that:

$$\frac{\partial L}{\partial q_i} = \frac{\partial u(\mathbf{q}^*)}{\partial q_i} - \lambda^* p_i = 0, i = 1, 2, .., n$$

and

$$\frac{\partial L}{\partial \lambda} = \mathbf{p}\cdot\mathbf{q} - y = 0$$

Characterization of the Solution – cont.

- The condition $\mathbf{p} \cdot \mathbf{q} - y = 0$ simply says that the consumer spends all her income
  - This follows from the assumption of strict monotonicity.
- If the Lagrange parameter λ^* is strictly positive, the remaining conditions can be rewritten as

$$\frac{\partial u(\mathbf{q}^*)/\partial q_j}{\partial u(\mathbf{q}^*)/\partial q_k} = \frac{p_j}{p_k}$$

  for all $j = 1, 2, .., n$ and $k = 1, 2, .., n$
  - This says that at the optimum, the marginal rate of substitution between any two goods must be equal to the ratio of the goods' prices

Marshallian Demand Function

- The solution to the consumer's problem depends on the parameters $\mathbf{p}$ and y of that problem
- The corresponding function $\mathbf{q}^* = \mathbf{q}(\mathbf{p}, y)$ is known as the ordinary, or Marshallian demand function
- This demand function summarizes all relevant information about consumer demand behavior

Properties of the Marshallian Demand Function

- If our model of consumer behavior is correct, then demand behavior must display certain observable characteristics
- These can be used to test the theory on the one hand and to improve empirical estimates when the theory is taking for granted on the other hand
  1. Budget Balancedness: $\mathbf{p} \cdot \mathbf{q}(\mathbf{p}, y) = y$ holds for all $\mathbf{p}; y$
  2. Homogeneity of Degree Zero: $\mathbf{q}(\mathbf{p}, y)$ is homogeneous of degree zero in all prices and income
  3. The Slutsky matrix $[s_{ij}]$ is symmetric and semidefinite

$$s_{ij} = \frac{\partial q_i(\mathbf{p}, y)}{\partial p_j} + q_j(\mathbf{p}, y)\frac{\partial q_i(\mathbf{p}, y)}{\partial y}$$

Budget Balancedness and Homogeneity

- Budget Balancedness simply means that consumer spending exhausts income
- Homogeneous of degree zero in all prices and income means:

$$\mathbf{q}(t\mathbf{p}, ty) = \mathbf{q}(\mathbf{p}, y)$$

  for all $\mathbf{p}, y, t > 0$
  - To prove that this property holds, it suffices to observe that the budget set remains unchanged when all prices and income are multiplied by $t > 0$
  - Only relative prices and real income affects demand behavior

Slutsky Substitution Effects

- Rewriting

$$s_{ij}(\mathbf{p}, y) = \frac{\partial q_i(\mathbf{p}, y)}{\partial p_j} + q_j(\mathbf{p}, y)\frac{\partial q_i(\mathbf{p}, y)}{\partial y}$$

  as

$$\frac{\partial q_i(\mathbf{p}, y)}{\partial p_j} = s_{ij}(\mathbf{p}, y) - q_j(\mathbf{p}, y)\frac{\partial q_i(\mathbf{p}, y)}{\partial y}$$

  provides the following interpretation:
  - Total effect of an increase in price p_j on the demand for good i
  - Income effect of an increase in price p_j on the demand for good j
  - Substitution effect of an increase in price p_j on the demand for good i
- $s_{ii}(\mathbf{p}; y)$ is the (familiar?) substitution effect of a change in price p_i on the demand for good i

Negative Semidefiniteness and Symmetry of the Slutsky Matrix: What does it Mean?

- An $n \times n$ matrix A is negative semidefinite if for all vectors $z \in R^n$, $\mathbf{z}^T A\mathbf{z} \leqslant 0$
  - In particular, if A is negative semidefinite its diagonal elements satisfy $a_{ii} \leqslant 0$
  - So, negative semidefiniteness of the Slutsky matrix is a generalization of the observation that the own-price substitution effects $s_{ii}(\mathbf{p}; y)$ are negative
- The symmetry of the Slutsky matrix means that the substitution effect of an increase in the price of good j on the demand for good i is identical to the substitution effect of an increase in the price of good i on the demand for good j

Further interpretation

- Negativity of the own-price substitution effects $s_{ii}(\mathbf{p}; y)$ implies the law of demand
- Recall the following definitions:
  - A good is normal if its consumption increases when income increases, holding prices constant
  - A good is inferior if its consumption decreases when income increases, holding prices constant
- Law of Demand: A decrease in the own price of a normal good causes the quantity demanded of the good to increase

Integrability Theorem

- Budget balancedness and symmetry imply homogeneity
- The Integrability Problem: we have a demand function, is there any utility function that generates it?

Theorem

A continuously differentiable function $\mathbf{q} : R_{++}^{n+1} \rightarrow R_{+}^{n}$ *is the demand function generated by some increasing quasiconcave utility function if it satisfies budget balancedness, symmetry and negative semidefiniteness*

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- Hence we can consider linear demand functions

Data

- To empirically estimate market demand, we need data
- Ideally, this data will have come from a controlled experiment
- This way we are assured that our data is a random sample
- We conduct the following experiment to collect demand data on good Q
  1. We randomly sampled N consumers from the population
  2. For each consumer i, we measure the consumer's income y_i, and then confront the consumer with a randomly selected price p_i
  3. We then observe consumer i's buying behavior q_i
- The experiment will result in a random sample of data

$$\{q_i, p_i, y_i : i = 1, 2, ...N\}$$

- To derive an econometric model through which we can estimate market demand, we translate the economic model into a statistical model
- The quantities q_i that we observe in the data are random variables
- In general, we can decompose a random variable into two parts:
  1. a systematic part μ
  2. an unobservable error ε_i^D
- The systematic part of q_i is its expected value $\mu = E[q_i]$
- The unobservable error ε_i^D is the stochastic part of q_i $var[q_i] = var[\varepsilon_i^D] = \sigma^2$

- We use economic theory to give us our predictions about q_i.
- For example

$$E[q_i] = \alpha_0 + \alpha_1 p_i + \alpha_2 y_i$$

- The error ε_i^D is an IID "demand shock" which captures how consumer decisions may differ from the average
- Thus, our econometric model for demand is

$$q_i = \alpha_0 + \alpha_1 p_i + \alpha_2 y_i + \varepsilon_i^D, \varepsilon_i^D \sim IID(0, \sigma^2)$$

Interpretation

- $\alpha_0, \alpha_1, \alpha_2$ are the parameters from the consumer utility function
- Moreover, the price elasticity and the income elasticity of demand are

$$\eta_p = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} = \alpha_1 \frac{p_i}{q_i}$$

$$\eta_y = \frac{\partial q_i}{\partial y_i} \frac{y_i}{q_i} = \alpha_2 \frac{y_i}{q_i}$$

- Given the random sample nature of our data $\{q_i, p_i, y_i : 1 = 1, 2, ...N\}$ our econometric model $q_i = \alpha_0 + \alpha_1 p_i + \alpha_2 y_i + \varepsilon_i^D, \varepsilon_i^D \sim IID(0, \sigma^2)$ has certain useful properties.
- Assumptions:
  1. $E[\varepsilon_i^D] = 0$
  2. $E[\varepsilon_i^D p_i] = 0$, i.e. p_i is independent from ε_i^D
  3. $E[\varepsilon_i^D y_i] = 0$, i.e. y_i is independent from ε_i^D
- They are satisfied given the random sample properties
- Thus the OLS estimator is unbiased and consistent, α is identified by OLS
- Note the importance of having a random sample
  - It is the random sampling assumption through which we can assert that Ass.1., 2., 3. are satisfied
  - And it is this moment condition which allows us to identify α by OLS