Delegation as a signal: implicit communication with full cooperation

Joanna Franaszek*

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Abstract

I examine the issue of implicit signaling of inexpressible type through delegation in a communication game with perfectly aligned preferences, two-sided private information, and communication frictions. The model is analyzed in the context of decision-making in teams. A principal (manager) consults an agent (expert) to choose one of two actions. The manager has some tacit knowledge, which he cannot communicate and may acquire some imperfect, costly signal about the state of the world. After observing the signal, he may choose the action or delegate to the expert, who observes the state of the world perfectly. Even if the principal's information acquisition and the signal are unobservable, the delegation, combined with private information, allows the agent to extract some signal about the tacit information. I show that for a large class of parameters there exists an equilibrium, in which the expert (upon delegation) can correctly understand "cues"Â and tailor the action to the manager's needs. In particular, the agent's decision may be non-monotone in the state of the world.

1 Introduction

How can one party pass some information to their partner without using direct communication? One way is to signal through the choice of action. However, choosing not to act – here, meaning delegation of authority – can also give the other party a signal about

*SGH Warsaw School of Economics, jfrana@sgh.waw.pl

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our private information. Such ”indecisiveness” might turn out to be beneficial even if the delegating agent is relatively well informed about the decision-relevant variable.

In this paper, I examine a general principal–agent model in the context of decision–making in an organization. In contrast to the existing literature, I assume the agent to be fully altruistic towards the principal – thus there is no intrinsic divergence of preferences. However, players face a problem of two-sided private information and difficulties in communicating their signal to the other party. Only the principal knows his private preference parameter, while it is the agent who perfectly observes the state of the world. The principal might privately obtain an informative but costly and imperfect signal about the state and then either decide about the action or delegate the decision to the agent. In the equilibrium, the agent correctly anticipates which principal types choose to be informed and under what conditions they would further decide to delegate authority, even though she observes nothing but the delegation decision. In particular, positive investment in an informative signal is correctly anticipated to come from a principal type that is intermediate – not leaning towards any of the actions. Moreover, the informed types delegate authority only if their acquired signal is inconclusive. This fact, along with the agent's knowledge of the true state of the world, allows her to correctly guess the most likely range of types and, in turn, adjust her choice of action to better tailor the principal's needs. In other words, the principal implicitly (and imperfectly) signals his type with the delegation, and the sophisticated agent is able to understand the cue. This results in a nontrivial equilibrium, in which the agent's final decision (upon delegation) is non-monotone in the state of the world.

The rest of the paper is structured as follows: first, I describe a motivating example and place the framework within a specific type of principal-agent situation with nondivergent preferences. Then I briefly summarize links with the broader literature on delegation, imperfect signals, and language frictions. In the next section, I introduce the model and present some results. I show that while signaling is present in the more general version of the model, more restrictive conditions are needed to guarantee the existence of the non-monotone equilibrium. Finally, I draw directions for future research.

1.1 Motivating example

Consider a manager (he) consulting an expert (she) to discuss two possible actions – e.g. whether to accept or reject a project, depending on some state of the world $x \in [0, 1]$. The manager has some private preference t, which may be interpreted as a decision cutoff: if $x > t$, the manager prefers one action (say, accept), while if $x < t$, the other action (reject) is preferred. Moreover, I assume the utility is continuous, which implies that around the cutoff $x = t$ the principal is close to indifference. The information about t is a form of tacit knowledge, which is difficult, or even impossible to express in terms of any language – an assumption that may be considered reasonable in the case of preferences.

The manager may privately obtain a costly informative signal about the state of the world. Neither the investment in information nor the realization itself is observed by the expert. After observing the signal, the manager may choose the preferred action herself or delegate the decision to the expert, who observes x perfectly. I assume the expert's utility coincides with the manager's and there is no inherent divergence of preferences – however, the exact value of t is known only to the manager, while the expert has some prior belief with a probability density function $g(t)$.

Suppose the expert ex-ante expects the manager types to be distributed uniformly, with half of the managers preferring acceptance and the other half – rejection. If the manager was not allowed to acquire information and delegated the choice to the expert, she would naturally choose rejection for $x < \frac{1}{2}$ and acceptance for $x > \frac{1}{2}$. The altruistic expert has no incentive to suggest acceptance for any $x > \frac{1}{2}$. The situation changes, however, if the manager is allowed to obtain an informative signal. We will assume that the signal is binary $s \in \{0, 1\}$, with $P(s = 1|x)$ increasing in x.

Consider an expert who is encountered by a manager with relatively low $x < \frac{1}{2}$, i.e. an indication for rejection for an ex-ante average type t. The expert believes the principal might have obtained an informative signal and, therefore, infers that an average informed manager *should* know that rejection is his best choice. Yet, the principal still prefers to delegate authority. The expert might then correctly infer that the delegation decision is more likely to come from a manager with low t, as types with $t \approx x$ find it most difficult to make an informed decision.[1] Thus, the expert's posterior belief about t becomes correctly

[1]The delegation decision comes from a principal whose signal was inconclusive, like in Li and Suen (2004); Garfagnini et al. (2014)

”slanted”. Based on her belief, the expert’s strategy would change as well – she is less likely to recommend a rejection whenever the decision is delegated. Moreover, if the expert believes the delegating manager to be quite well–informed, the slant might even overturn the ex-ante recommended action. This might look surprising, but, in fact, is desirable from the manager’s perspective – the delegation allows him to improve the final choice even more than making an informed decision.

To make the intuition more concrete, let us choose a particularly simple form of the signal: $\hat{p}(x) = 1_{\{x \geq \frac{1}{2}\}}$. Such a signal is the “most informative” of symmetric binary signals, as it gives the precise location of x. Let us assume the utility is of the form $a(x-t)$, where $a \in \{0,1\}$ is a binary action, moreover $g(t) = U[0,1]$ and the cost of obtaining the signal is $c = \frac{1}{36}$.

I claim that the equilibrium is as follows: the principal acquires an informative signal whenever $t \in \left[\frac{1}{4}, \frac{3}{4}\right]$. Moreover, for $t \in \left(\frac{5}{12}, \frac{7}{12}\right)$ he would retain the authority, choosing an action in line with the signal. For $t \in \left[\frac{1}{4}, \frac{5}{12}\right]$ he would delegate if the signal is $s = 0$ and for $t \in \left[\frac{7}{12}, \frac{3}{4}\right]$ he would delegate if the signal is $s = 1$. The expert chooses $a = 1$ (upon hearing delegation) if and only if $x \in \left[\frac{1}{3}, \frac{1}{2}\right] \cup \left[\frac{2}{3}, 1\right]$, and thus, his strategy is non-monotone in the state of the world.

Let us first analyze the expert’s strategy, taking the principal’s choice as given. Upon hearing delegation, the expert anticipates $t \in \left[\frac{1}{4}, \frac{5}{12}\right] \cup \left[\frac{7}{12}, \frac{3}{4}\right]$. However, since x is observed by the expert, she knows exactly which value of the signal must have been realized.[2] Therefore, if $x < \frac{1}{2}$ he knows delegation comes from $t \in \left[\frac{1}{4}, \frac{5}{12}\right]$ and for $x > \frac{1}{2}$ the delegating type must be $\left[\frac{7}{12}, \frac{3}{4}\right]$. Her posterior belief is then $E(t|D,x) = \frac{1}{3}$ for $x < \frac{1}{2}$ and $E(t|D,x) = \frac{2}{3}$ for $x > \frac{1}{2}$. The expert chooses $a = 1$ whenever $x \geq E(t|D,x)$, which leads exactly to the profile above. As for the principal, a formal derivation is a bit more tedious, but the intuition is simple: everyone apart from extreme types gets cheap information. Middle types follow the signal and retain authority; “leaning” types follow the signal if it confirms their prior choice and delegate whenever they become uncertain about the optimal action, i.e. whenever s is close to their type.

[2] Frankel and Kartik (2019) describe a two-dimensional information that is *muddled* into one-dimensional action, so “any observed action will generally not reveal either dimension”. Here, the information is de-muddled – the agent uses his knowledge of x to separate “low types with low signal” from”high types with high signal”. Such a phenomenon would arise only imperfectly in the general model, where the probability of any signal realization is non-degenerate.

The simple example gives a flavor of the main result of the paper: if an interesting equilibrium with cues exists for the simplest signal structure $\hat{p}(x) = 1_{\{x \geq \frac{1}{2}\}}$, then it would also exist for some family of signals in the neighborhood of $\hat{p}$, that I describe formally in assumption 1.

It must be noted that the equilibrium described above is unique in this setup. Observe that even without the expert, principal types $t \in [1/4, 3/4]$ would still find it optimal to acquire information (and a possibility to delegate would weakly enhance their incentives, whatever the expert strategy is). Moreover, at least types $t = 3/4$ and $t = 1/4$ would delegate (as they are indifferent between retaining and delegating authority), so the set of informed-and-delegating types is nonempty. Since the signal depends on x only through $1_{\{x \geq \frac{1}{2}\}}$, $E(t|x) = E(t|s)$ is a function that takes at most two values $E(t|s = 1)$ or $E(t|s = 0)$. The expert's optimal choice is $a^D(x) = 1_{\{x>1/2 \wedge x>E(t|s=1)\}} + 1_{\{x<1/2 \wedge x>E(t|s=0)\}}$. Suppose that $x < \frac{1}{2}$. Then $a^D(x) = 1_{\{x>x_0\}}$ (possibly for $x_0 \geq \frac{1}{2}$). In such a case, all types $t \in [1/4, \frac{x_0 + \frac{1}{2}}{2}]$ would delegate, thus resulting in $E(t|D, x) = \frac{1}{8} + \frac{x_0 + \frac{1}{2}}{4}$. Since it must also hold that $x_0 = E(t|D, x)$, then the only solution is $x_0 = 1/3$. Similar reasoning applies to $x > \frac{1}{2}$, resulting in a profile determined above.

1.2 Related literature

The model is connected to a few strands of literature. The first area is a vast literature on delegation versus communication, in line with the idea of Dessein (2002). However, while most of the delegation literature (see e.g.Li and Suen (2004); Alonso and Matouschek (2008); Garfagnini et al. (2014); Deimen and Szalay (2019) concentrate on strategic incentives with divergent, or, at most, correlated preferences of the decision–maker and the expert(s), I focus solely on communication frictions and make a bit unpopular assumption that the two parties share same preferences. This assumption, a bit similar to the one in Dewatripont (2006), not only allows me to examine information transmission in isolation but also changes the players' incentives. The players want to exchange as much information as possible – both through direct communication and through signaling – but face language constraints in communication. This is what distinguishes my model from other models of cooperation via communication, as Dziuda and Gradwohl (2015); Li et al. (2001).

My setup is similar to Garfagnini et al. (2014), with nondivergent preferences. Just like in their work, I observe that the act of delegation is informative about the agent's signal. However, in my setup, where the information acquisition is endogenous (contrary to Garfagnini et al. (2014)) the delegation becomes informative also about the principal's preference type. This guarantees the existence of a sophisticated equilibrium with cues about one's type.[3]

The cues sent in the model have a flavor of signaling as in Spence (1973). However, while in classic signaling models the correlation between the unobserved type and the signal is explicit i.e. the costs of signaling are lower for types with higher productivity. In my setup, the correlation between the unobserved preference and the state, observed by the agent arises endogenously and only through a complex mechanism of correlated informative signals and optimal decisions. Moreover, the principal's signaling through delegation is only to a little extent driven by signaling incentives. In fact, even if the agent was blind to signaling and chose the action only according to the state and the prior belief about the principal type, there would be still room for delegation for at least some principal types. However, the model is much more interesting if the agent can "infer beliefs from actions" (see Arieli and Mueller-Frank (2017)).

The transmission of complex information falls into the strand of the literature on knowledge dissemination. As noticed e.g. by Boldrin and Levine (2005), the mere *availability* of information does not make the information *accessible* to a person, who may lack the expertise to interpret it. Communication is, therefore, costly (see also Austen-Smith (1994); Hedlund (2015); Eso and Szentes (2007); Gentzkow and Kamenica (2014) for other models of explicitly costly information transmission). Moreover, complex knowledge takes years to build and cannot be easily and costlessly transmitted. In fact, some information – such as the principal's preferences toward alternative treatments – might be impossible to verbalize. This *tacit knowledge*, as defined in Polanyi (1966), can only be transmitted through non-verbal cues, as in this model, where it is signaled in the equilibrium choices.

[3] I use the word "cues", as in Dewatripont and Tirole (2005), however, the meaning of the term is very different. While in Dewatripont and Tirole (2005) sending cues is a substitute for a potentially costly communication, in my model it is a way to implicitly signal one's type in the presence of language constraints

2 Model

2.1 Utility

To examine the communication frictions in isolation, I assume the agent to be fully altruistic towards the principal. In other words, the agent and the principal have the same utility function, which depends on the principal's state of the world and the choice of treatment. In particular, I assume the utility depends on (x, t, a) where $x \in [0, 1]$ is the state, $t \in [0, 1]$ is the principal's preference type and $a \in \{0, 1\}$ is a binary action (e.g. reject or accept). Note that x is only observed by the agent, and t is only observed by the principal. The two parameters can only be imperfectly transmitted to the other party.[4] The prior distribution of x and t, are, respectively $U[0, 1]$ and G_t with some continuous, full support density $g(t)$, and this is common knowledge. I assume $g(t)$ is symmetric around an axis $t = \frac{1}{2}$. This assumption is not crucial in establishing the existence of an equilibrium but significantly helps in understanding the main contribution of the paper.

For expositional simplicity, I will normalize the payoff from action $a = 0$ to 0 and assume that u takes a linear form

$$u(x, t, a) = a(x - t) \text{ for } a \in \{0, 1\},$$

which is just enough to generate interesting results.[5]

2.2 Communication frictions

I assume there are two types of communication frictions, regarding both information about t and x. As for the preference parameter t, I assume it is a form of Polanyi's tacit knowledge, which cannot be explained in terms of language and is therefore impossible to communicate either via cheap–talk or any form of disclosure. The only information about t

[4]The assumption that x and t are some numbers in the unit interval is just for expositional simplicity. We could imagine a setup in which x is a point in some multidimensional abstract space. The space is separated into two compact areas and in each of them one treatment is preferred to the other. The type t would then be a boundary (e.g. a line, a plane, a manifold) between the two. However, the continuity and monotonicity conditions that are stipulated for unit interval need to be adapted to the multidimensional setting.

[5]It is worth noticing that the setup could be easily generalized to some non-linear utility, as long as it is strictly increasing in the distance between x and t. In particular, for any strictly increasing function h,

that the agent can have comes from his beliefs regarding the principal's observed actions.[6]

Moreover, the expert knowledge is complex and therefore hard to transmit precisely to the non-expert manager. Therefore, any information about x is necessarily imperfect. Moreover, understanding at least some information about x requires some mental or monetary cost, which could be interpreted as a cost of translating technical terms into everyday language, effort in communication, time devoted to explanations, etc. I assume that the choice of information acquisition is binary, i.e. the principal may decide to acquire an informative signal about the state of the world x at a cost c or remain uninformed (equivalently: receive an uninformative signal) at no cost. The information might come from some private source, but a setup could also be used to analyze the case in which the principal acquires information from the expert himself. In this interpretation, the expert tries to communicate the state of the world and the manager may exert zero or positive (namely, c) effort in understanding it. The expert can observe neither the effort choice nor the final realization of the signal (i.e. what the principal understood from his explanation). To simplify the analysis I assume that an informative signal is binary, i.e. $s \in \{0, 1\}$ and could be interpreted as a recommendation of action, such that action 1 is recommended more often for higher states.[7] In particular, I shall assume that for any x the probability of signal $s = 1$, denoted by $P(s = 1|x) = p(x)$, belongs to a very specific family of S-shaped signals.

Assumption 1. *The probability of signal $s = 1$, denoted by $p(x)$, satisfies:*

1. *$p(x)$ is increasing in x, with $p(0) = 0$ and $p(1) = 1$.*
2. *The signaling technology is symmetric around $x = \frac{1}{2}$, i.e. $p(x) = 1 - p(1 - x)$. In particular, $p(\frac{1}{2}) = \frac{1}{2}$.*
3. *$p(x)$ is S-shaped with $\int_0^{1/2} p(x) < \epsilon(\tau)$.*

The first assumption is quite straightforward. The second means the signal is "fair", in a sense it treats low states and high states symmetrically. The third assumption ensures that $p(x)$ is sufficiently steep around $x = \frac{1}{2}$, in other words, the signal discriminates well between states that are higher and smaller than the average. This assumption is crucial in

[6]While the assumption that t is inexpressible is quite strong, it serves as a useful benchmark and a departure from large family of models with perfect communication.

[7]Note that such a recommendation is *not* cheap-talk.

establishing an equilibrium with the agent's non-monotone choice of action. The steepness of the signal structure ensures that the principal who chooses to acquire information is well-informed at least about the direction of recommendation. While he lacks full information about the value of x, an informed principal acquires a quite precise signal about the state being higher or lower than the average. If given such precise information, the principal still prefers to delegate, the choice of delegation becomes a strong signal about the principal's type and allows the agent to infer similarly strong beliefs about the possible range of t.[8]

To examine some technical requirements links the signal and the distribution of t, I would denote $E_g(t|t \in [\frac{1}{4}, \frac{1}{2}]) = \tau$ and describe some of the results in relation to τ. The parameter τ is a proxy of the concentration of $g(t)$ around the mean $Eg(t) = \frac{1}{2}$. Note that the agent's problem of decision under delegation is more pronounced if τ is small i.e. there is a significant measure of principals with strong preferences towards one of the treatments.[9]

I will show that there exists an equilibrium in which the principal chooses a symmetric strategy, but the understanding of the cue makes the agent's belief (correctly) leaning to one side. For some range of signals, the slant is sufficient to change the ex-ante optimal action, which results in an interesting non-monotone action profile. The family of signals becomes larger as τ gets smaller, i.e. as $g(t)$ becomes less concentrated around its mode.

2.3 Timing

The timing of the model is as follows:

1. Nature draws x (learned by the agent) and t (learned by the principal).

[8] One might wonder whether the signal structure might be more complex, for example continuous. The answer is yes, but with serious restrictions. Preliminary results about continuous choice of investment in information suggest that one needs to be more careful when checking incentive compatibility conditions for any arbitrary investment in information. Moreover, while only weak assumptions about informativeness of a signal are needed to obtain some slant in the agent's belief, the slant effect is strong enough only under quite specific conditions. In particular, any signal structure that is to lead to the non-monotonic choice of action must be sufficiently informative about the direction of x.

[9] Distributions highly concentrated around the mean would have large (i.e. close to $1/2$) values of τ, while introducing some dispersion would decrease the parameter. Notice that from the agent's perspective the extreme types $t < 1/4$ or, symmetrically, $t > 3/4$ are irrelevant, as they never delegate. Thus, the "dispersion" parameter τ is only calculated for non-extreme types.

2. The principal chooses to acquire an informative signal about x at cost $C = c$ (or an uninformative signal at cost $C = 0$).

3. After observing $s|x$ the principal decides to:

   (a) retain the authority,

   (b) delegate the decision to the agent.

4. The chosen decision-maker chooses an action $a \in \{0, 1\}$.

5. The utility $u(x, t, a) - C$ is realized.

I would show that by the choice of information and then delegation, the principal implicitly signals his type t and signal realization s. However, contrary to classic signaling models, his choices are not pure signals. In fact, the principal decides to acquire information primarily to improve his likelihood of the right choice and the agent's strategy only slightly enhances the principal's incentives. This makes the acquisition choice particularly robust. However, the implicit signaling feature would be crucial in examining the equilibrium behavior.

3 Equilibrium choices

The equilibrium concept is a Perfect Bayesian Equilibrium. Denote the principal's strategy as $(C(t), \sigma(s,t), a(s,t))$ with the investment in information $C : [0,1] \to \{0, c\}$, allocation of authority $\sigma(s,t) : [0,1]^2 \to \{D, P\}$ and the choice of action $a^P(s,t) \to \{0,1\}$. Denote the agent's response after delegation as $a^D(x) : [0,1] \to \{0,1\}$ and his posterior belief as $\mu(s, t|\sigma, x)$. Since we are mainly interested in the agent's posterior belief about the type, let us also denote $g(t|D, x) := E_s\mu(s, t|D, x)$.

I shall propose a specific form of an equilibrium and prove its existence. In the putative equilibrium, the principal strategy is symmetric around $t = \frac{1}{2}$. Extreme types of principals do not acquire information and choose the action by themselves (since they are already certain about their decision). The middle types, who are ex-ante close to indifference, either acquire information – if it is cheap – and make an informed decision or remain uninformed – if the information is expensive – and delegate the authority. Somewhat leaning types chose conditional delegation i.e. they acquire an informative signal and delegate the decision only if it is "inconclusive". The expert agent chooses an action, based on his belief about t (and her knowledge of x), choosing $a^D(x) = 1$ if $x > E_\mu(t|D,x)$ and $a^D = 0$ if $x < E_\mu(t|D,x)$. More specifically, the agent chooses $a = 1$ if and only if $x \in [\bar{x}, \frac{1}{2}] \cup [1-\bar{x}, 1]$, with $\bar{x} \leq \frac{1}{2}$. Observe that for $\bar{x} = \frac{1}{2}$ the strategy coincides with the trivial "ex-ante" profile; however, in the more interesting case of $\bar{x} < \frac{1}{2}$ the strategy is non-monotone in the state of the world.

3.1 Action choice

Let us now proceed with formal backward analysis for a general signal structure. Going backward, we shall start with the action choice. If the principal makes the decision himself, he has no strategic interaction to consider. Therefore, he would choose $a = 1$ if $E(x|s) > t$ and $a = 0$ otherwise. Notice that for an uninformative (zero cost) signal $E(x|s) = Ex = \frac{1}{2}$. On the other hand, after an informative signal, the expectation $E(x|s)$ depends on $s \in \{0, 1\}$. Then $E(x|s = 1) > \frac{1}{2} > E(x|s = 0)$. However, the general rule $a = 1 \Leftrightarrow E(x|s) > t$ does not change.

If the decision was delegated to the agent, he would choose one action over another based on the value of x. From the principal's point of view, the optimal agent's choice of action $a^D(x)$ can be determined using the posterior belief about t, which also depends on the agent's information x. In the equilibrium, the principal can correctly anticipate the agent's posterior belief $E(t|D,x)$ and takes $a^D(x)$ as given. Assume $a^D(x) = 1_{\{\bar{x} \leq x \leq \frac{1}{2}\}} + 1_{\{1-\bar{x} < x \leq 1\}}$ as described above.

Once the action choice in the last step is determined, we can proceed with a backward analysis of the principal's incentives in each step. Through this section, let us denote by $V(t, C, \sigma)$ the expected value of an information investment choice C and allocation of authority σ (which may be conditional on observed s).

3.2 Allocation of authority

First, let us assume that the principal did not invest in an informative signal and therefore has only prior belief on x. Such a principal would decide to delegate rather than retain authority if[10]:

$$V(t, 0, D) \geq V(t, 0, P) \Leftrightarrow \int_0^1 (x - t) a^D(x) dx \geq \max\left(\frac{1}{2} - t, 0\right). \quad (1)$$

[10] I implicitly assume that the indifferent principal chooses delegation.

I shall denote the types who choose uninformed (therefore, unconditional) delegation by Ω_{UD}.

Claim 1. Suppose t does not acquire information. Then $t \in \Omega_{UD}$ iff $(1-t) \in \Omega_{UD}$ and if $\Omega_{UD} \neq \emptyset$ then $\frac{1}{2} \in \Omega_{UD}$.

A (very simple) proof of this claim and all subsequent can be found in the Appendix.

Now, consider the case with an informative signal. The principal would only invest in a signal if he is willing to use it. We can therefore exclude the case in which the principal would choose an action that goes against the signal realization, as such a principal would prefer not to acquire information at all. Indeed, if a principal observes s either he would choose $a = s$ immediately or delegate the decision to the agent. The principal prefers delegation to retainment if:

$$V(t,c,D|s) \geq V(t,c,P|s) \Leftrightarrow E(x-t|s,\ D) \geq E(x-t|s,P)$$

$$\begin{cases} \int_0^1 (x-t)p(x)a^D(x)dx \geq \int(x-t)p(x) & \text{if } s=1, \\ \int_0^1 (x-t)(1-p(x))a^D(x)dx \geq 0 & \text{if } s=0. \end{cases}$$

With simple algebra and an observation that $\frac{\int xp(x)1_{\{a^D=i\}}dx}{P(a^D=i)} = E(x|s,a^D = i)$, the conditions could be summarized as:

$$\begin{cases} E(x|s,a^D=0) \leq t & \text{for } s=1, \\ E(x|s,a^D=1) \geq t & \text{for } s=0. \end{cases} \tag{2}$$

Claim 2. For any informative signal realization s, there exists a range of principal types Ω^s_{CD}, who prefer to delegate the authority, conditionally on being informed.

To get an intuition about the result, notice that for any signal realization at least type $t = E(x|s)$ would find it strictly profitable to delegate, as conditional on his information, he is indifferent between the two actions. Thus, he may benefit by delegating to the possibly better–informed agent. By continuity, there exists a range of types around $t = E(x|s)$ who would also prefer delegation. The intuition that relatively large (small) types delegate whenever the signal is large (small)- and thus the type is implicitly correlated with signal – would be crucial in understanding the equilibrium communication. Full proofs of all the claims can be found in the Appendix.

Claim 3. The sets Ω^0_{CD} and Ω^1_{CD} are symmetric around an axis $t = \frac{1}{2}$ and disjoint i.e. $\Omega^0_{CD} = [\underline{t}, \bar{t}]$ and $\Omega^1_{CD} = [1-\bar{t}, 1-\underline{t}]$ for some $\underline{t} < \bar{t} < \frac{1}{2}$.

The symmetry of delegation decision is a direct result of the symmetry of a^D and the signal $s|x$ around $x = \frac{1}{2}$. This implies the symmetry of the delegation decision. No type delegates for both signal realizations, as such a type would prefer to deviate to not acquiring signal at all (and delegating immediately).

3.3 Information acquisition

Take the strategies in the second period described in the previous subsection as given. Going one step back, the principal needs to decide whether to invest in an informative signal or not. The expected value of the decision in the second step is the maximum of the two possible options (delegation or retainment) for both levels of investment. Therefore, the principal would choose to acquire an informative signal:

$$E_s \max\{V(t,c,D|s), V(t,c,P|s)\} - c \geq \max\left\{V(t,D,0), V(t,P,0)\right\}.$$

Lemma 1. *The set of types who acquire information Ω_c is symmetric around $t = \frac{1}{2}$. Extreme types $t = 0$ and $t = 1$ (and their neighborhood) never acquire information.*

For a given signal distribution p there exists two upper bounds ψ, ϕ, such that if $c \in (0, \psi)$ then $\Omega_c \ni \frac{1}{2}$ and there exist types who acquire information and choose according to their signal. In such a case the set Ω_{UD} is empty. If $c \in [\psi, \phi]$, then all types who acquire information delegate conditionally on their signal $\Omega_c = \Omega_c \cap \left(\Omega^0_{CD} \cup \Omega^1_{CD}\right)$. If $c > \phi$, nobody acquires information and the game is trivial.

The two lemmas are simply summarized in Figure 1. Extreme types close to $t \in \{0, 1\}$ have such strong preferences towards one of the treatments that they do not feel the need to invest in information. If the cost of information is small, all "medium types" acquire information and some of them choose conditional delegation. Notice that by Claim 3, informed-and-delegating types form two disjoint intervals, therefore types close to $t = 1/2$ choose to make a decision by themselves. This is pictured in the left panel of Figure 1.

If the cost of the signal is big enough, the types close to $t = \frac{1}{2}$ are hit by a "median principal curse".[11] Notice that type $t = \frac{1}{2}$ finds it ex-ante most difficult to choose between the two actions, therefore he has an incentive to acquire information. However, since the information is (ex-ante) symmetric, type $t = \frac{1}{2}$ would expect it to be inconclusive and costly. Therefore, he would rather delegate to a perfectly informed agent.

[11]Notice that for a symmetric distribution $median = mean$.

Figure 1: The investment in information and delegation choice for $c < \psi$ (left) and $\psi \leq c \leq \phi$ (right).

Note that by Lemma 1, there are no other switches in the principal strategy than those in Figure 1.

3.4 Agent's strategy and beliefs

Given the (known) informativeness of the signal, the agent expects the principal to delegate whenever $t \in \Omega^0_{CD} \cup \Omega^1_{CD} \cup \Omega_{UD}$. The agent optimally chooses actions, taking into account his expectation about the type. Formally, the agent chooses $a^D(x) = 1$ for $x \geq E(t|D,x)$ and $a^D(x) = 0$ for $x < E(t|D,x)$. Notice however that the formula $E(t|D,x)$ is not constant and is a function of x. Denote:

$$\beta(x) = E(t|D,x) = \frac{\int tg(t|D,x)dt}{\int g(t|D,x)dt},$$

where $g(t|D,x)$ is the interim agent's belief given the observed x and the expected principal's strategy $\{\sigma(s) = D\} \Leftrightarrow t \in \Omega_{CD} \cup \Omega_{UD}$. For a given x, the posterior belief about the type distribution $g(t|D,x)$ would typically *not* have full support, neither will it be symmetric around $t = 1/2$. The agent knows x, therefore, he can correctly infer what are the probabilities of acquiring a specific signal. In particular, the interval "closer" to x is more likely than the other. Thus, if agent observes x (say, $x > \frac{1}{2}$) and delegation, then he correctly infers that the most likely signals are those "close to" x, and such signals are indecisive for types t "close to" x. Therefore, his posterior belief about types is skewed towards x. Formally:

$$g(t|D,x) = \begin{cases} \frac{g(t)(1-p(x))}{A(x)} & \text{for } t \in \Omega^0_{CD} \\ \frac{g(t)p(x)}{A(x)} & \text{for } t \in \Omega^1_{CD} \\ \frac{g(t)}{A(x)} & \text{for } t \in \Omega_{UD} \\ 0 & \text{for all other } t \end{cases}$$

where $A(x) = (1-p(x)) \int_{\Omega^0_{CD}} g(t)dt + p(x) \int_{\Omega^1_{CD}} g(t)dt + \int_{\Omega_{UD}} g(t)dt$. Note that since g is symmetric then $\int_{\Omega^1_{CD}} g(t) = \int_{\Omega^0_{CD}} g(t)$, therefore $A(x) = \int_{\Omega^0_{CD}} g(t)dt + \int_{\Omega_{UD}} g(t)dt$.

Lemma 2. *Assume $g(t)$ is symmetric around an axis $t = \frac{1}{2}$. Then in any equilibrium, C, σ are symmetric around an axis $t = \frac{1}{2}$, but the agent's cutoff function (i.e. the a posteriori expected principal type) $E(t|D,x)$ is **not** symmetric around an axis $x = \frac{1}{2}$. Instead, $\beta(x) = E(t|D,x)$ is weakly increasing in x (with $\beta(\frac{1}{2}) = \frac{1}{2}$) and $\beta(x) + \beta(1-x) = 1$.*

The statement of the Lemma may not look as exciting as it is. To fully understand its value, notice first that if the agent's posterior belief about the distribution of t was symmetric around $t = \frac{1}{2}$, the function $\beta(x)$ would be constant and equal to $\frac{1}{2}$, independently of x. However, the agent's belief in the equilibrium is skewed towards the "correct" t, even though the principal's strategies are symmetric. This phenomenon can be only sustained whenever the agent observes x, because then, given distribution $p(x)$ he can determine more likely signal realizations and, using their equilibrium association with t, infer what are more likely values of t. The agent not only correctly anticipates the information acquisition choice, but also exploits the correlation of the agent's and principal's signals. The correlation along with the equilibrium delegation decision allows the agent to infer the most likely range of t, even though t is never explicitly signaled. However, the important issue is whether the "slant" in posterior beliefs is strong enough to induce the agent to change the a priori optimal actions. For an S-shaped $p(x)$ this is exactly the phenomenon that may arise.

Denote by $\tilde{\tau} := \frac{\int_{\Omega^0_{CD}} tg(t)dt + \int_{\Omega_{UD}} tg(t)dt}{A}$, that is, the expected type of delegating principals. Observe that if $\Omega_{UD} = \emptyset$ then $\tilde{\tau} < \tau := E(t|t \in [1/4, 1/2])$. However, in general $\tilde{\tau}$ is determined in the equilibrium – in particular, it depends on c.

Lemma 3. *If $p'(\frac{1}{2}) > \frac{1}{1-2\tilde{\tau}}$ (with a sufficient condition $p'(\frac{1}{2}) > \frac{1}{1-2\tau}$ if $c < \psi$) the agent's action profile follows a non-monotone pattern, that reflects his asymmetric belief upon observing delegation:*

$$a^D(x) = \begin{cases} 1 & \text{for } x \in \left[\bar{x}, \frac{1}{2}\right] \cup [1-\underline{x}, 1], \\ 0 & \text{otherwise}, \end{cases} \quad \text{for some } \bar{x} < \frac{1}{2}$$

Otherwise, the agent's action profile in equilibrium coincides with the "naive" one:

$$a^D(x) = \begin{cases} 1 & \text{for } x \in \left[\frac{1}{2}, 1\right], \\ 0 & \text{otherwise}. \end{cases}$$

The agent's choice is pictured in Figure 2. In equilibrium with cues, the agent recommends action $a = 0$ for x small (which is intuitive), but also for relatively big $x \in \left(\frac{1}{2}, 1 - \underline{x}\right)$. The second interval is the region in which the slant in posterior beliefs plays a dominant role. In particular, even though the agent knows x is relatively big a priori, the implicit signal coming from the recommendation makes him believe t is even bigger. Therefore action $a = 0$ is preferred. Such a nontrivial profile is only possible when the effect on the posterior beliefs induced by a signal and the delegation decision is strong enough. In particular, the marginal change in beliefs around $x = \frac{1}{2}$ must be large, namely $p'(\frac{1}{2}) > \frac{1}{1-2\tilde{\tau}}$. However, if this requirement is not satisfied, then even though the posterior belief is indeed skewed in the "right" direction, the perturbation is not strong enough to induce a switch from naive beliefs.

As a corollary from the Lemma 3, we can claim the following main result of the paper:

Theorem 1. *There exists a Perfect Bayesian Equilibrium of the game with implicit signaling of type through delegation. In such an equilibrium, the principal's strategy is symmetric around* $t = \frac{1}{2}$*, while the agent's strategy may be non-monotone in the state of the world. In particular, the equilibrium choices are as follows:*

1. *If* $c \leq \psi$ *then only the extreme principal remain uninformed (and retain their authority). The middle types acquire information and retain authority, while the "somewhat slanted" types delegate conditionally on the signal. If* $p'(\frac{1}{2}) > \frac{1}{1-2\tilde{\tau}}$*, the agent responds with a non-monotone action profile.*

2. *If* $c \in [\psi,\ \phi]$ *then only the somewhat slanted types acquire information. The extreme types remain uninformed and retain the authority. The middle types remain uninformed and delegate authority. If* $p'(\frac{1}{2}) > \frac{1}{1-2\tilde{\tau}}$ *(which now is a stronger condition than above, as* $\tilde{\tau}$ *depends on c), the agent responds with a non-monotone action profile.*

3. *If* $c > \phi$ *no principal type acquires information. Types* $t \in [\frac{1}{4},\ \frac{3}{4}]$ *delegate and the remaining types retain authority. The agent's strategy is trivial, as no information about the signal is transmitted by delegation.*

The theorem describes signal families, for which the agent's strategy becomes non-monotone in action. The intuition is that the signaling function should indeed resemble a letter 'S' and be steep around $p(1/2)$. Notice that in the introductory example we examined

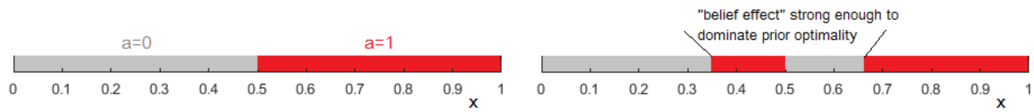

Figure 2: The action profile chosen by the agent in equilibrium if $p'(\frac{1}{2})$ is small (left) and large (right)

a limit case with ”infinite”[12] steepness and perfectly flat tails. Lemma 3 and theorem 1 explain how this extreme signal structure could be generalized and adapted to continuous signal functions.

4 Summary

In this paper, I examine a principal-agent model with two-sided private information and no conflict of interest in the context of agent–principal communication. I assume that the decision–relevant information is two–dimensional and that each dimension is observed just by one agent. I show that if the principal has access to a costly, but informative signal about the dimension known to the agent, the agent can not only anticipate which types would find it valuable to acquire information, but also correctly infer the cue about the principal type from his decision to delegate or retain the authority.

The model describes a phenomenon of non-verbal communication through equilibrium actions and demonstrates how the joint information about the principal’s strategies *and* the agent’s private signal allow the latter to make nontrivial conclusions about the type of the former. Such a phenomenon is only possible because the two parties obtain potentially correlated information about the same variable and the delegation choice gives implicit information about both the observed signal and its relation to the principal’s preferences. As a result, the agent’s belief is correctly slanted. Moreover, if the signal is S-shaped and sufficiently precise in distinguishing states $x > \frac{1}{2}$ from states $x < \frac{1}{2}$ the slant is strong enough to change the a priori optimal actions and there may arise an equilibrium, in which the agent’s action profile chosen upon delegation is non-monotone in the state of the world.

I focus on communication frictions and implicit signaling to stress that when the information is cheaply available, even if the players observe nothing but the apparent

[12]Formally, indefinite.

”indecisiveness” of the other party, a correct inference about their preferences and strategies can still be made. This implicit communication through delegation helps the players to coordinate on the preferred outcome.

5 Declaration of interest

The author declares that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix: Proofs

Proof of Claim 1: Inequality in (1) could be rewritten as:

$$\int_0^1 xa^D(x) \geq \max\left(\frac{t}{2}, \frac{1-t}{2}\right).$$

It is now clear that if the condition holds for t, it also holds $1-t$. Also, the RHS is minimized by $t = \frac{1}{2}$, so if any t satisfies the condition, so does $t = \frac{1}{2}$.

Proof of Claim 2: Assume that a^D follows the putative pattern. Intuitively, $a^D = 1$ must be chosen more often for higher states, so:

$$E(x|s, a^D = 0) < E(x|s) < E(x|s, a^D = 1), \tag{3}$$

since

$$E(x|s) = E(x|s, a^D = 0) \cdot P(a^D = 0) + E(x|s, a^D = 1) \cdot P(a^D = 1). \tag{4}$$

Consider t varying from 0 to 1. As t increases up to $E(x|s)$, it must hit a point where $\underline{t}^s = E(x|s, a^D = 0)$ and for all $t \in [\underline{t}^s, E(x|s)]$ the first inequality of (2) is satisfied. Similarly, for $t \searrow E(x|s)$ the second inequality of (2) is satisfied, and as t increases to 1, there exists a point $\bar{t}^s$ such that for any $t > \bar{t}^s$ the delegation is no longer preferred. Therefore, for $t \in [\underline{t}^s, \bar{t}^s] =: \Omega_D^s$ delegation is preferred, while for $t \notin [\underline{t}^s, \bar{t}^s]$ the principal prefers to retain authority. It is important to notice that $[\underline{t}^s, \bar{t}^s] \ni E(x|s)$ and since the delegation decision is made after learning s, the interval differs with the realization of s.

Proof of Claim 3: Assume t satisfies condition (2) for $s = 1$ and we need to show that $1-t$ satisfies it for $s = 0$. Without loss of generality, assume t satisfies the first inequality of (2), i.e.

$$E(x|s = 1) > t \text{ and } E(x|s = 1, a^D = 0) \leq t$$

Then I need to prove that $1-t$ satisfies inequality:

$$E(x|s = 0, a^D = 1) \geq 1 - t.$$

To prove this, it is enough to show that $E(x|s = 0, a^D = 1) = 1 - E(x|s = 1, a^D = 0)$. I shall use double symmetry of all the ingredients:

$$E(x|s=0,a^D=1)=\frac{\int x(1-p(x))1_{x\in\{a^D=1\}}}{\int 1-p(x)1_{x\in\{a^D=1\}}}=\frac{\int xp(1-x)1_{x\in\{a^D=1\}}}{\int p(1-x)1_{x\in\{a^D=1\}}}=$$
$$=\frac{\int(1-y)p(y)1_{y\in\{a^D=0\}}}{\int p(y)1_{y\in\{a^D=0\}}}=1-\frac{\int yp(y)1_{y\in\{a^D=0\}}}{\int p(y)1_{y\in\{a^D=0\}}}=1-E(x|s=1,a^D=0).$$

Therefore, if $t\in\Omega^1_{CD}$ then $1-t\in\Omega^0_{CD}$.

We might also notice that by more general considerations, the two intervals must be disjoint. If there exists some $t\in\Omega^0_{CD}\cap\Omega^1_{CD}$ who would delegate for any signal, then he would rather not acquire the costly information at all.

Such considerations allow us to use a somewhat simpler notation. Define $[\underline{t},\bar{t}]:=\Omega^0_{CD}$ then $\Omega^1_{CD}=[1-\bar{t},1-\underline{t}]$.

Proof of Lemma 1: There exist four possible strategies: uninformed decision $(C,\sigma)=(0,P)$, informed decision (c,P), uninformed delegation $(0,D)$, and informed delegation (c,D). Recall that by Claim 3 delegation for an informed agent is always conditional on the signal and is chosen only for a signal realization closer to t, while uninformed delegation is unconditional by definition. Moreover, if the principal finds it optimal to choose informed decision, it must be the case that the chosen actions are different for different signal realizations and consistent with them, more specifically $a^P=s$.

I shall analyze the payoffs of all the above strategies and try to determine what is the optimal profile given generic t. Since the problem is symmetric around $t=\frac{1}{2}$, I shall assume explicitly that $t\geq\frac{1}{2}$. Then:

$$V(t,0,P)=\max\left(\frac{1}{2}-t,0\right)=0$$
$$V(t,0,D)=\int_0^1(x-t)a^D(x)=\frac{3}{8}-\left(\frac{1}{2}-\bar{x}\right)^2-\frac{t}{2}$$
$$V(t,c,P)=P(s=0)\cdot 0+P(s=1)\int_0^1(x-t)2p(x)dx-c=\int_0^1 xp(x)dx-\frac{t}{2}-c$$
$$V(t,c,D)=P(s=0)\cdot 0+P(s=1)\int_0^1(x-t)2p(x)a^D(x)dx-c=\int_0^1(x-t)p(x)a^D(x)dx-c$$

I shall analyze how the optimal strategy changes with t moving from $\frac{1}{2}$ to 1 using five observations:

1. For $t=1$ uninformed decision $V(t,0,P)$ dominates any other strategy.

2. Either $V(t,c,P) \geq V(t,0,D)\ \forall t$ (for c small) or $V(t,c,P) < V(t,0,D)\ \forall t$ (for c big).

3. $V(t,c,D) > V(t,c,P)$ for t sufficiently big.

4. $V(t,c,D) > V(t,0,D)$ for t sufficiently big.

5. There exist t such that $V(t,c,D)$ is optimal, in particular, there are always types who acquire information.

The first observation is trivial: as $t = 1$ (or close to 1) all the payoffs become negative, apart from $V(t,0,p)$. The second observation stems from the fact, that both formulas are of the form $-\frac{t}{2} + constant$, so in comparison, only the constant matters. In particular:

$$V(t,0,D) \leq V(t,c,P) \Leftrightarrow \frac{3}{8} - \left(\frac{1}{2} - \bar{x}\right)^2 \leq \int_0^1 xp(x)dx - c$$

$$V(t,0,D) \leq V(t,c,P) \Leftrightarrow c \leq \underbrace{\int_0^1 xp(x)dx - \frac{3}{8}}_{<0} + \left(\frac{1}{2} - \bar{x}\right)^2 =: \psi \tag{5}$$

The necessary condition being:

$$\psi = \left(\frac{1}{2} - \underline{x}\right)^2 - \frac{3}{8} + \int_0^1 xp(x)dx \geq 0 \tag{6}$$

I shall claim that whenever $\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x)$ is sufficiently small, the inequality (6) holds. Namely, there exists a function $\epsilon(\tau)$ such that when $\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x) < \epsilon(\tau)$, then the inequality above is guaranteed. For the purpose of clarity, this part of the proof is moved below in the Appendix, under Lemma 4.

The third observation is a bit more subtle. Since $a^D(x)$ is an indicator function $V(t,c,D)$ is a "truncated" version of integral in$V(t,c,P)$, in which some values of the function $(x-t)p(x)$ are replaced by 0. Such a transformation may only be beneficial if the values replaced by 0 were negative. The function $(x-t)p(x)$ is negative for $x < t$, so the bigger t is, the more attractive is $V(t,c,D)$ relative to $V(t,c,P)$. Therefore $V(t,c,D) > V(t,c,P)$ for t sufficiently big. Similar reasoning applies to the fourth statement. $V(t,c,D)$ and $V(t,0,D)$ differ only by a weighting function and a constant c. The function $p(x)$ is between 0 and 1 and thus places relatively small weight on negative values of $(x-t)$, thus strongly diminishing their effect on the integral (while the effect of positive values is only slightly attenuated). In particular, notice that the limit of $V(t,c,D) - V(t,0,D)$ as $t \to 1$ is:

$$\int_0^1 (1-x)(1-p(x))a^D(x)dx - c > 0 \text{ for } c \text{ small.}$$

Observe, that if c satisfies inequality (5), then c also satisfies:

$$\int_0^1 (1-x)(1-p(x))a^D(x)dx - c \geq \int_0^1 (1-x)(1-p(x))a^D(x)dx - \int_0^1 xp(x)dx + \int_0^1 xa^D(x) =$$
$$= \int_0^1 p(x)(1-x)(1-a^D(x))dx > 0.$$

Therefore, as long as $c < \psi$, $V(t,0,D)$ is never chosen. In this case, types close to $1/2$ chose $V(t,c,P)$, bigger types switch to $V(t,c,D)$ and extreme types switch to $V(t,0,P)$.

It is a bit more difficult to deal with a slightly bigger c, that does *not* satisfy (5). Then the middle type would choose $(0,D)$ instead of (c,P). However, there are still some types, who acquire information. More specifically, for $t_0 > \frac{1}{2}$ satisfying $\int_0^1 (x-t_0)a^D = 0$ the optimal choice is (c,D), as long as c is not too big. Indeed, as $E(x|s=1) \leq E(x|s=1, a^D(x)=1$ and $E(x|a^D(x)=1) \leq E(x|s=1, a^D(x)=1$, then at least for t_0 it must be that $V(c,D,t_0)+c > 0$. By continuity, there exists an upper bound ϕ such that if $c < \phi$ at least someone acquires information.

If $c > \phi$ and nobody acquires information, the equilibrium is trivial, as no significant information is transmitted through delegation.

Proof of Lemma 2

Recall the definition of the expected type (upon delegation):

$$\beta(x) = E(t|D,x) = \int_0^1 tg(t|D,x)dt.$$

With:

$$g(t|D,x) = \begin{cases} \frac{g(t)(1-p(x))}{A} & \text{for } t \in \Omega_{CD}^0, \\ \frac{g(t)p(x)}{A} & \text{for } t \in \Omega_{CD}^1, \\ \frac{g(t)}{A} & \text{for } t \in \Omega_{UD}, \end{cases}$$

where $A = \int_{\Omega_{CD}^0} g(t)dt + \int_{\Omega_{UD}} g(t)dt$, independent of x. To see that $\beta(\frac{1}{2}) = \frac{1}{2}$, observe that $g(t|D,\frac{1}{2})$ is a symmetric density function, so the expected value with respect to t is $\frac{1}{2}$. Moreover, notice that $\beta(x)$ is simply an affine transformation of $p(x)$ that preserves symmetry around a point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

$$\beta(x) = \frac{1}{A}\left[p(x)\int_{\Omega^1_{CD}} tg(t)dt + (1-p(x))\int_{\Omega^0_{CD}} tg(t)dt + \int_{\Omega_{UD}} tg(t)dt\right].$$

$$\beta(x) = \left[p(x)\frac{\left(\int_{\Omega^1_{CD}} tg(t)dt - \int_{\Omega^0_{CD}} tg(t)dt\right)}{A} + \frac{\int_{\Omega^0_{CD}} tg(t)dt + \int_{\Omega_{UD}} tg(t)dt}{A}\right].$$

Denote by $\tilde{\tau} := \frac{\int_{\Omega^0_{CD}} tg(t)dt + \int_{\Omega_{UD}} tg(t)dt}{A}$, that is, the expected type of a principal who chooses delegation. Since both sets $\Omega^0_{CD} \cup \Omega^1_{CD}$ and Ω_{UD} are symmetric around $t = \frac{1}{2}$, it is easy to show that $\frac{\left(\int_{\Omega^1_{CD}} tg(t)dt - \int_{\Omega^0_{CD}} tg(t)dt\right)}{A} = 1 - 2\tilde{\tau}$. Therefore:

$$\beta(x) = (1-2\tilde{\tau})p(x) + \tilde{\tau}$$

In particular, $\beta(x)$ is increasing, convex on $\left[0, \frac{1}{2}\right)$ and concave on $\left(\frac{1}{2}, 1\right]$ and symmetric around $\left(\frac{1}{2}, \frac{1}{2}\right)$ i.e. $\beta(x) = 1 - \beta(1-x)$.

Proof of Lemma 3:

The proof is simple and follows directly from properties of $\beta(x)$ derived in the proof of Lemma 2 above. Recall that the agent chooses $a^D(x) = 1$ if $x > \beta(x)$ and $a^D(x) = 0$ otherwise. From Lemma 2 we already know $\beta(x)$ is increasing and crosses line $id(x) = x$ at least in $x = \frac{1}{2}$. Since it's an affine transformation of $p(x)$ and $\lim_{x\to 0}\beta(x) > 0$ (which implies $\lim_{x\to 1}\beta(x) < 1$), then it crosses line $id(x) = x$ at most three times in $(0,1)$. I will show that if $\beta'(x) > 1$ then $\beta(x) - x = 0$ has exactly three solutions in $(0,1)$ and define

$$\underline{x} = \min\{x : \ \beta(x) = x\}. \tag{7}$$

Let us start with the limit. This is simple: observe that whatever x is, if delegation was observed, it must have come from a type $t \in \Omega_{CD} \cup \Omega_{ND}$. Then

$$\forall x \ E(t|D,x) \geq \min \Omega_{CD} \cup \Omega_{UD} = \underline{t} \ \Rightarrow \ \lim_{x\to 0} E(t|D,x) \geq \underline{t} > 0.$$

For the derivative, recall that:

$$\beta'(x) = p'(x)(1-2\tilde{\tau}).$$

$$\beta'(x) > 1 \Leftrightarrow p'(x) > \frac{1}{1-2\tilde{\tau}} =: \alpha(c). \tag{8}$$

Assume c is small, namely $c \leq \psi$, as defined in inequality (5). For such a c, no types choose to delegate conditionally. Then $\tilde{\tau} = E(t|t \in \Omega^0_{CD})$ and $\frac{1}{1-2\tilde{\tau}} < \frac{1}{1-2\tau}$, therefore as long as $p'(x) > \frac{1}{1-2\tau}$, the existence of $\underline{x} < \frac{1}{2}$ is guaranteed, regardless of c. For $c > \psi$, however, no useful upper bound exists.

Lemma 4. *If* $\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x)dx \leq \phi(a) := \frac{(1-2a)(3-5a)-a\left(\sqrt{a^2+2(1-2a)(3-5a)}-a\right)}{16(3-5a)^2}$ *then inequality (6) holds.*

Proof. Denote $r = \left(\frac{1}{2} - \underline{x}\right)$. Recall that by definition of $\underline{x}$ in (7):

$$\underline{x} = p(\underline{x})(1-2\tilde{\tau}) + \tilde{\tau}.$$

Notice that $p(x)$ might be considered a cumulative distributive function for some continuous symmetric unimodal distribution Z. It is easy to determine that:

$$Var(Z) = \int_0^1 x^2 p'(x)dx - \frac{1}{4} \stackrel{parts}{=} \left(1 - \int_0^1 2xp(x) - \frac{1}{4}\right) = 2\left(\frac{3}{8} - \int_0^1 xp(x)dx\right).$$

On the other hand, since Z is symmetric, we can write:

$$Var(Z) = \int_0^1 \left(\frac{1}{2} - x\right)^2 p'(x)dx = 2\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right)^2 p'(x)dx \stackrel{parts}{=} 4\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x)dx.$$

Since p is concave on $\left(0, \frac{1}{2}\right)$, it can be bounded from above by piecewise linear function, using the property $p(\underline{x}) = \frac{\underline{x}-\tilde{\tau}}{1-2\tilde{\tau}}$:

$$\begin{aligned}\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x)dx &= \int_0^{\underline{x}} \left(\frac{1}{2} - x\right) p(x)dx + \int_{\underline{x}}^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x)dx \leq \\ &\leq \int_0^{\underline{x}} \left(\frac{1}{2} - x\right) \frac{p(\underline{x})}{\underline{x}} xdx + \int_{\underline{x}}^{\frac{1}{2}} \left(\frac{1}{2} - x\right) \frac{x-\tilde{\tau}}{(1-2\tilde{\tau})}dx = \\ &= \frac{1}{48(1-2\tilde{\tau})}\left(-8\tilde{\tau}\underline{x}^2 + 12\tilde{\tau}\underline{x} - 6\tilde{\tau} + 1\right) = \frac{1}{48(1-2\tilde{\tau})}\left(1 - 2\tilde{\tau}\left(4r^2 + 2r + 1\right)\right).\end{aligned}$$

A sufficient condition for inequality (6) to hold is therefore:

$$\frac{1}{12(1-2\tilde{\tau})}\left(1 - 2\tilde{\tau}\left(4r^2 + 2r + 1\right)\right) \leq 2r^2$$

It is satisfied whenever $r \geq \frac{1}{4(3-5\tilde{\tau})}\left(\sqrt{\tilde{\tau}^2 + 2(1-2\tilde{\tau})(3-5\tilde{\tau})} - \tilde{\tau}\right)$ or, alternatively, if $Var(Z) < 4\epsilon(\tilde{\tau})$, where $\epsilon(\tilde{\tau}) = \frac{1}{12}\frac{(1-2\tilde{\tau}(4r^2+2r+1)}{(1-2\tilde{\tau})}$ evaluated at $r = \frac{1}{4(3-5\tilde{\tau})}\left(\sqrt{\tilde{\tau}^2 + 2(1-2\tilde{\tau})(3-5\tilde{\tau})} - \tilde{\tau}\right)$. After a bit of tiresome algebra, we get the required sufficient condition to be:

$$\int_0^{\frac{1}{2}} \left(\frac{1}{2} - x\right) p(x) \leq \frac{(1-2\tilde{\tau})(3-5\tilde{\tau}) - \tilde{\tau}\left(\sqrt{\tilde{\tau}^2 + 2(1-2\tilde{\tau})(3-5\tilde{\tau})} - \tilde{\tau}\right)}{16(3-5\tilde{\tau})} =: \epsilon(\tilde{\tau}).$$

Notice that $\tilde{\tau}$ is determined in equilibrium. However, if $\Omega_{UD} = \emptyset$ then $\tilde{\tau} = E(t|t \in \Omega^0_{CD}) < \tau$ and since ϵ is increasing, then the condition with τ instead of $\tilde{\tau}$ is stronger (and sufficient). To get some more intuition, check Figure 3 for a plot of an upper bound on $Var(Z)$ and notice that since that for an arbitrary unimodal distribution we only have $Var(Z) \leq \frac{1}{12}$, the bound is non-trivial and indeed necessary. □

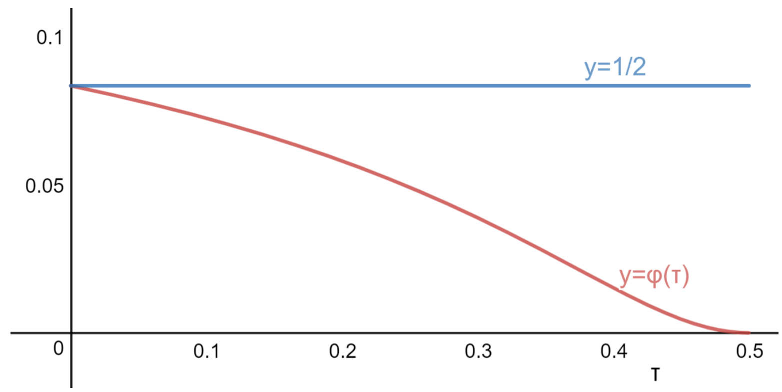

Figure 3: Upper bound on variance $\phi(\tau)$ (red) that guarantees the existence of an interesting equilibrium

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