Communication - cheap talk models

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Communication

Common market (and non-market) activity used for

- information transmission
- signalling
- coordination etc.

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- coordination etc.

Some simple models

Battle of the sexes

- Players: husband and wife go *independently* to either Opera or Football game
- husband prefers Opera, wife prefers Football
- …but most of all: they prefer to meet!

		Husband: Opera	Husband: Football
Wife	Opera	$(1, 2)$	$(0, 0)$
	Football	$(0, 0)$	$(2, 1)$

Battle of the sexes

- classic version: no communication
- two PSNE: (Opera, Opera), (Football, Football)
- one MSNE: $((\frac{1}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{2}{3}))$
- inefficient: in MSNE players miscoordinate in 5/9 cases!
- expected return: 2/3 (less then going to less prefered event all the time!)

Battle of the sexes

Now, allow for communication!

- suppose wife says: ”I’m going to a Football game”
- what can the husband think?
  - if she is really choosing F, would she want me to believe it? yes
  - if she is going to O, would she want me to believe F? no
  - $\Rightarrow$ message is self-signaling!
  - if she thinks she persuaded me she’s going to F, does she have an incentive to go to F? yes
  - $\Rightarrow$ message is self-commiting!

Language

If anything can be said, meaning of messages is an equilibrium outcome:

- Battle of the sexes: ’natural’ equilibrium with language in which if S says ’Opera’, R believes that S is indeed going to an opera
- but also: an equilibrium in which if S says ’Opera’, is in fact going to Football game **and it is perfectly understood by S**
- ...the meaning of a word ’Opera’ is only determined by S’s messages and (consistent) R’s beliefs & actions
- ’messages have only meaning in equilibrium’
- **language is a coordination device**
- for now: stick to ”natural language” more

Prisoner's dillema

- Joint school project done in pairs
- each player can exert high or low effrot

		Barney High	Barney Low
Ann	High	$(3, 3)$	$(1, 4)$
Ann	Low	$(4, 1)$	$(2, 2)$

Prisoner’s dillema

- low effort is a dominant strategy!
- unique NE: (L, L)
- Assume Ann says ”I will exert high effort”
  - not self-committing
  - not self-signaling

Stag hunt

- Artemis and Calliope go for a hunt
- they might go for an easy prey (rabbit) individually, or a difficult prey (stag)
- if both hunt for stag, they do very well
- if one hunts for stag, fails
- rabbit is a 'safe option'

		Calliope: Stag	Calliope: Rabbit
Artemis	Stag	$(9, 9)$	$(0, 7)$
	Rabbit	$(7, 0)$	$(6, 6)$

Prisoner’s dillema

- coordination game!
- role for communication
- Artemis says ”I will hunt for a stag”
  - it is self-committing (if A thinks C believes it, she would indeed hunt stag)
  - but **not** self-signaling (A wants to persuade C even if she hunts rabbit)
- not credible
- *empirically.... who knows?*

Sender-Receiver game

Setup:

- Sender (=Agent) and Receiver (=Principal)
- Sender posseses some information, relevant to Receiver's decision
- Sender sends a message
- Receiver:
  - takes into account S incentives
  - forms some beliefs about use of messages by S
  - forms beliefs about ”meaning of messages”

Example

Principal-agent and sender-receiver problem:

- Max wants to work at a bank. He can be a trader (high pay, high stress) or a researcher (low pay, low stress)
- Max's stress handling level is unobservable
- Payoffs depend on Max type:
- 1st assumption: Max and bank have 'common' preferences

		Bank	
		Trader	Researcher
Max stress resistance	High	$(4, 4)$	$(2, 2)$
	Low	$(0, 0)$	$(2, 2)$

- Max's possible message "I'm high type"/"I'm low type" is self-signalling
- communication resolves info asymmetry
- (possibly cheaper than costly signaling)

Example

Principal-agent and sender-receiver problem:

- Max wants to work at a bank. He can be a trader (high pay, high stress) or a researcher (low pay, low stress)
- Max's stress handling level is unobservable
- Payoffs depend on Max type:
- 2nd assumption: incentives diverge!

		Bank	
		Trader	Researcher
Max stress resistance	High	$(4, 4)$	$(2, 2)$
	Low	$(\mathbf{3}, 0)$	$(2, 2)$

- Max's possible message "I'm high type" is not credible anymore!
- (meaningful) communication not possible!
- (at least if lying is possible...)

Example

Principal-agent and sender-receiver problem:

- 3rd assumption: in-between
- Suppose Max has three stress handling levels with same prior probability
- Bank has three possible jobs: trader, sales representative and researcher

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress resistance	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
	Medium	$(3, 1)$	$(2, 2)$	$(2, 1)$
	Low	$(0 - 3)$	$(1, -1)$	$(2, 1)$

- let's break it down...

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress	High	$(4,5)$	$(3,3)$	$(2,1)$
resistance	Medium	$(3,1)$	$(2,2)$	$(2,1)$
	Low	$(0-3)$	$(1,-1)$	$(2,1)$

- does Max want to reveal truthfully only his high type, i.e. $m(H) = H, m(L) = m(M) =$"M or L"

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
resistance	Medium	$(3, 1)$	$(2, 2)$	$(2, 1)$
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- does Max want to reveal truthfully only his high type, i.e. $m(H) = H, m(L) = m(M) =$"M or L"
- **No.** Medium Max prefers to be a trader, while bank would hire "non-high" Max as researcher.

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
resistance	Medium	$(3, 1)$	$(2, 2)$	$(2, 1)$
	Low	$(0 - 3)$	$(1, -1)$	$(2, 1)$

- does Max want to truthfully reveal his low type? $m(L) = L, m(H) = m(M) =$"M or H"

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
resistance	Medium	$(3, 1)$	$(2, 2)$	$(2, 1)$
	Low	$(0 - 3)$	$(1, -1)$	$(2, 1)$

- does Max want to truthfully reveal his low type? $m(L) = L, m(H) = m(M) =$"M or H"
- **Yes.** Low Max prefers to be a researcher, so does the bank. "Non-low" Max would be hired my the bank as trader, as

$$Eu_B(trader|M \wedge H) = 3 > 2.5 = Eu_B(sales\ rep.|M \wedge H).$$

- Max always gets his preferred job!

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
resistance	Medium	$(3, 1)$	$(2, 2)$	$(2, 1)$
	Low	$(0 - 3)$	$(1, -1)$	$(2, 1)$

- does Max want not to say anything? $m(L) = m(H) = m(M) =$"not going to tell you". The bank then offers a job of sales rep. Is it credible?

Example

		Bank's offer		
		Trader	Sales Rep.	Researcher
Max stress resistance	High	$(4, 5)$	$(3, 3)$	$(2, 1)$
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- does Max want not to say anything? $m(L) = m(H) = m(M) =$"not going to tell you". The bank then offers a job of sales rep. Is it credible?
- **Yes,** under some specific beliefs... Need to formalize the game!

Sender-receiver formal game

Bayesian game:

- $t \in T$ is Max type with commonly known prior $p(t)$;
- Max chooses messages $m(t) \in M$ (M= set of all available messages)
- bank chooses $j(m)$, job offers from set of all jobs J upon hearing message m
- bank beliefs conditional on message m are $p(t|m)$

Perfect Bayesian Equlibrium:

- $m(t)$ maximize Max expected payoff given $j(m)$
- $j(m)$ maximize bank's expected payoff given $m(t)$ and $p(t|m)$
- $p(t|m)$ is calculated using Bayes rule whenever possible
- (whenever possible = for nodes reached with positive probability $\Rightarrow$ off-equilibrium beliefs arbitrary!)

Out-of-equlibrium beliefs

Need to specify out-of-equilibrium beliefs

- suppose bank upon hearing ’unexpected’ message has belief that it is the medium type
- it’s perfectly ok!
- ...and makes ”babbling” equilibrium (in which Max says ”I’m not going to tell you my type”) possible
  - in the babbling equilibirum, bank offers Max a sales representative job
  - Max has no incentive to deviate, because any other message would also give him a sales rep. job
- **Much more general result**: with no further restrictions on messages/believes, **a bablling equlibrium always exists** in a communication game

Cheap-talk model (Crawford, Sobel)

Crawford, Sobel (1982), Strategic information transmission

- +4000 citations (as of May 2019)
- key model in communication theory
- tractable + very nice equilibrium description + comparative statics

CS setup

- Sender and Receiver
- Sender observes private signal $m \in [0, 1]$ with prior dist $f(m)$
- Sender sends a message n to the Receiver to induce some action
- the Receiver takes action $y \in R$, that influences utility
- Receiver has utility $U(y, m)$, Sender $U(y, m, b)$, b is preference parameter
    - unique best response: for each y $\exists! m$ such that $U_y^R(y, m) = 0$
    - concavity: $U_{yy}^i(.) < 0$
    - sorting: $U_{ym}^i(.) > 0$

Equilibrium concept

Bayesian Nash Equilibrium

- signaling rule (chosen by S): $q(n|m)$ such that if m is observed, message n is sent with probability $q(n|m)$
- action rule (chosen by R) $y(n)$ such that if n is observed, y is chosen
- the rules must satisfy:
  - R's rule optimal given S's. if n' taken with positive prob. in m, then:

$$n' \in \arg\max_n U^S(y(n), m, b), \text{ given } y(n)$$

  - S's rule optimal given R's

$$y(n) \in \arg\max_y \int_0^1 U^R(y, m) p(m|n),$$

where $p(m|n) = \frac{q(n|m) f(m)}{\int_0^1 q(n|m) f(m)}$

General idea of solution

- in a canonical example $U^R(m, y) = -(y - m)^2$ and $U^S(m, y) = -(y - (m + b))^2$
- thus, players preferences are 'close', but differ by b
- can communication be credible?
    - if Sender says his 'bliss point' $n = m + b$, then Receiver would discount that and consider state $y = n - b$. But the Sender would try to fool him and say $n = m$... and so on
    - precise messages indeed non-credible!
    - but if we allow for some slack...
    - ...cheap-talk becomes meaningful

Preview of the equlibrium

- Partition:
  - in the equilibrium, Sender would partition signal space
  - message of type "*m* is between 0.1 and 0.2"
  - there is a **finite** number of partitions
  - size of 'finest' partition depends on preference discrepancy b
- Credibility
  - messages are credible!

Equilibrium construction

Let $y^i(m, b) = \arg\max U^i(y, m, b)$

Lemma

If $y^S(m, b) \neq y^R(m)$ $\forall m$ then $\exists \epsilon > 0$ such that if u and v are actions induced in equilibrium, $|u - v| > \epsilon$. Further, the set of actions induced in equilibrium is finite

Recall the example $U^R = -(y - (m + b))^2$, $U^S = -(y - m)^2$. Then $y^R(m) = m$ and $y^S(m, b) = m + b$ and Lemma holds.

Proof of Lemma

- Let $y^S(m, b) > y^R(m)\ \forall m$ (as in canonical example); in particular $\exists \epsilon\ y^S - y^R > \epsilon$
- Suppose type m_u induce u and type m_v induces v, with $v > u$
- Then $U^S(u, m_u, b) \geq U^S(u, m_v, b)$ (by optimality)
- Also $U^S(v, m_v, b) \geq U^S(v, m_u, b)$
- by continuity, $\exists \bar{m}$ such that $U^S(u, \bar{m}, b) = U^S(v, \bar{m}, b)$
- Since $U^S_{yy}(.) < 0$ and $U^i_{ym}(.) > 0$, then:
    - $u < y^S(\bar{m}, b) < v$
    - u not induced by any $m > \bar{m}$
    - v not induced by any $m < \bar{m}$
    - $\Rightarrow u < y^R(\bar{m}) < v$

$$u < y^R(\bar{m}) < y^R(\bar{m}) + \epsilon < y^S(\bar{m}, b) < v \Rightarrow v - u > \epsilon$$

Equilibrium construction cont'd

Let $0 = a_0 < a_1 \ldots < a_N = 0$ denote a partition of $[0, 1]$ into N subintervals. If $m \in [a_i, a_{i+1}]$, the BR of Receiver is:

$$\bar{y}(a_i, a_{i+1}) = \arg\max \int_{a_i}^{a_{i+1}} U^R(y, m) f(m) \forall n \in [a_i, a_{i+1}]$$

Theorem

Suppose b is such that $y^S(m, b) \neq y^R(m)$ $\forall m$. *Then* $\exists$ *a positive integer* $N(b)$ *such that for any N such that* $1 \leq N \leq N(b)$, *there exists an equilibrium with N distinct messages i.e.* $n = n_i$ *if* $n_i \in (a_i, a_{i+1})$ *and* $n_i \neq n_j$ *for* $i \neq j$.

Moreover, any equlibrium is essentially equivalent to one of this class.

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Moreover, any equlibrium is essentially equivalent to one of this class.

Leading example

$$U^S(y, m, b) = -(y - (m + b))^2, \; U^R(y, m) = -(y - m)^2$$

Optimal choice:

$$y^S(m, b) = m + b, \; y^R(m) = m$$

Best response to $n_i \in [a_i, a_{i+1}]$

$$\bar{y}(a_i, a_{i+1}) = \frac{a_i + a_{i+1}}{2}$$

Leading example cnt'd

In the equilibrium, the partition is constructed so that the Sender is indifferent between $\bar{y}(a_i, a_{i+1})$ and $\bar{y}(a_{i-1}, a_i)$ exactly at a_i:

$$\left(\frac{a_i + a_{i+1}}{2} - (m+b)\right)^2 = \left(\frac{a_{i-1} + a_i}{2} - (m+b)\right)^2$$

This only holds if they differ in sign:

$$\left(\frac{a_i + a_{i+1}}{2} - (a_i+b)\right) = -\left(\frac{a_{i-1} + a_i}{2} - (a_i+b)\right)$$

That is:

$$a_{i+1} - a_i = a_i - a_{i-1} + 4b$$

That is, each step size must increase by $4b$!

Importance of b

As the partition depends on b, it also determines the maximal partition size:

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{b}\right)^{1/2} \right\rceil$$

To at least have some information in (one) equlibrium, $b < \frac{1}{4}$. As $b \to 0$, scope for more and more information transmission!

$\Rightarrow$ if preferences are closer, parties have more meaningful communication, even in cheap-talk!

Illustration

Take $b = 1/40$. Then the maximal partition is of size 4 and intervals increase by 0.1

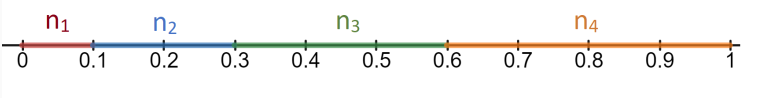

Figure 1: Most informative equilibrium partition

Sample messages: '0.05', '0.2', '0.45', '0.8'.

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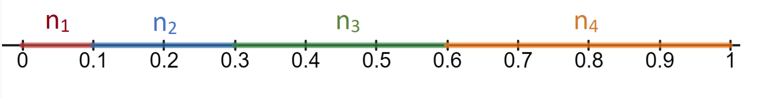

Figure 1: Most informative equilibrium partition

Sample messages: '0.05', '0.2', '0.45', '0.8'.

(But could be different - recall that in cheap-talk messages only have meaning in equilibrium) recollection

Summary

Seminal model of cheap-talk

- non-trivial general result
- but a very tractable example!
- **no restrictions** on message space (lying, babbling, balderdash)
- two players with different, but 'sufficiently' close objectives
- quite a lot of meaningful communication!
- **if there is a will, there is a way (to communicate)**

Appendix - language

Natural language

Assumption about ’natural language’ rule out ’non-intuitive’ equilibria:

- if R ever chooses action *a* (following some message), then in particular he would choose *a* after hearing *a*
- therefore, in any equilibrium if R goes to Opera, in particular he must do so after he hears message ’go to Opera’
- this is an equilibrium selection criterion
- helps to deal with ’real-life’ problem on how equlibria are reached
- (unsurprisingly) confirmed in multiple experiments

Go back to main presenation